SIMPLEX AND INTERIOR POINT METHODS FOR SOLVING BUDGETARY ALLOCATION LINEAR PROGRAMMING PROBLEM IN FOOD INDUSTRY

by

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LIST OF ABBREVIATIONS

LP	Linear Programming
IPM	Interior Point Method
SM	Simplex Method
SAR	Saudi Arabia Riyal

KAEDAH SIMPLEKS DAN KAEDAH LALUAN DALAM UNITUK PENYELESAIAN PERMASALAHAN PENGATURCARAAN LINEAR DALAM PERUNTUKAN BELANJAWAN MAKANAN INDUSTRI

ABSTRAK

Model pengoptimuman matematik mempunyai kriteria penting untuk mengoptimumkan pelbagai permasalahan linear industri yang terutamanya melibatkan pengmaksimuman keuntungan atau pengurangan kos. Terdapat pelbagai model matematik yang penting yang boleh digunakan untuk menyelesaikan pengaturcaraan linear untuk belanjawan dalam industri seperti kaedah simpleks dan Kaedah Laluan Dalam. Dalam kajian ini, dua model pengoptimuman matematik dibangunkan iaitu Kaedah Simpleks (KSO) dan Kaedah Laluan Dalam (KLD) untuk menyelesaikan permasalahan pengaturcaraan linear. Fungsi objektif bagi model pengaturcaraan linear adalah untuk memaksimumkan keuntungan dalam syarikat pembuatan makanan. Dengan merujuk kepada kekangan-kekangan yang ditentukan oleh syarikat dalam aspek kuantiti pembuatan, amaun perolehan dan keuntungan yang dirancang, kedua-dua model dibangunkan untuk memenuhi kesemua kekangan yang dikenakan untuk memperolehi bilangan optimum bagi kuantiti pembuatan yang perlu dikeluarkan dalam sebulan. Produk-produk yang dirujuk terutamanya melibatkan enam jenis produk yang utama iaitu Kacang Merah (K_1), Kacang Hijau (K_2), Kacang Kuda (K_3), Hamos (K_4), Kacang Putih (K_5) dan Kacang Besar (K_6) yang dikeluarkan oleh Syarikat Makanan LANA dalam tempoh 10 bulan pada tahun 2014. Hasil kajian menunjukkan bahawa nilai keuntungan optimum yang dihasilkan oleh model Kaedah Laluan Dalam adalah selari dengan hasil keputusan daripada model Kaedah Simpleks yang mana kedua-duanya menjanjikan kaedah model pengoptimuman untuk permasalahan pengaturcaraan linear dalam industri

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ABSTRACT

Mathematical optimization models have essential criteria for optimizing various industrial linear problems mainly involve the profit maximization or cost reduction. There are various significant mathematical models employed for solving budgetary linear programming in industries such as the Simplex and Interior Point Methods. In this study, two mathematical optimization models are developed namely Simplex Method (SM) and Interior Point Method (IPM) to solve linear programming problem. The objective function of linear programming model is to maximize the profit in food manufacturing company. By referring to the constraints that determined by the company in the aspect of production quantities, planned amount of profits and revenues, both models are developed to satisfy all constraints in order to obtain optimal number of product quantities to be produced monthly. The products that been referred mainly come from six type of products namely, Red Beans (K_1), Green Beans (K_2), Chick Peas (K_3), Hamos (K_4), White Beans (K_5) and Large Beans (K_6) from LANA Company for Food Ltd for a period of 10 months in the year 2014. The results indicated that the IPM method produced optimal profit values than its counterpart method and thus it's a promising optimization model for linear programming problem in industries.

CHAPTER 1 : INTRODUCTION

1.1 Introduction

This chapter is dedicated to the general context of this research. First, the theoretical background of the mathematical constrained models particularly the Simplex Method (SM) and the Interior Point Method (IPM) to solve Linear Programming (LP) optimization problems in budgetary planning is given. Afterwards, the research problem statement, research objectives and research scope are exposed. Finally, a brief summary of the research contributions as well as the organization of this dissertation are presented.

1.2 Background

In recent years, the advances in engineering, business research and computer technologies have extended the usage of mathematical models to solve wide range of linear problems. As a consequence, mathematical models are used extensively in almost all areas of decision-making including profit and budgetary planning, resources allocation, facility locations, process production and scheduling, machine and job scheduling, personnel, product mixes and inventory management. Mathematical models provide quicker, safer and cheaper approach to solve linear and decision-making problems than constructing and manipulating real systems (trial-and-error approach)(Maurya, Misra, Anderson, & Shukla, 2015).

Generally, the constrained optimization models are mathematical models that aims at finding the best solution for a given problem from a set of alternative solutions with respect to some evaluation criteria (constraints) and an objective function. A model represents the vital features of an object, system or problem without irrelevant details which can be characterised in mathematical forms using parameters, variables and objective functions. The most widely used mathematical model for solving various applied science problems is the Linear Programming (LP). The specific reason behind the wide applicability of LP is their simplicity, flexibility and ability to solve optimization problems as a system of linear inequalities (Fagoyinbo & Ajibode, 2010).

Over the past decades, mathematical computational and optimization models particularly Linear Programming (LP) have attempted to provide a systematic, quantitative ways to evaluate and select decisions for budgetary planning in industrial organizations (Hillier & Lieberman, 2001). Budgetary planning is an essential attribute in industries in which a decision on allocating the financial resources may influence the outcome of the profit, cost and the performance of the industry (Alluwaici, Junoh, Zakaria, & Desa, 2017; Khan, Bajuri, & Jadoon, 2011).

LP is a mathematical constrained method that is used to solve linear problems particularly to maximize profits or minimize costs subjected to constraints on capacities, resources, demands and supplies. LP has proven to be a tremendously powerful tool in both; mathematical theory and in modelling for real-world problems to select alternatives in multi-objective decision making problems (Yahya, Garba, & Ige, 2012). Nevertheless LP has been applied extensively in various fields such as operational research, engineering, production and operation management, finance, human resources, economics, marketing and distribution (Haddad, 2016).

Basically, any linear program constrained model composes of four main parts which are; decision variables, the model parameters, evaluation criterion (objective function) and a set of mathematical constraints. The constraints are mathematical inequalities of equalities which are used to define the solutions to an optimization or constrained mathematical model (A bichandani, Torabi, Basu, & Benson, 2015).

Apart from this, LP problems can be solved through the use of one or two algorithms: either Simplex Method (SM), or Interior Point Methods (IPM)s. The development and analysis of both SM and IPM algorithms are related with the LP method. In fact, the earliest algorithm used for solving LP problems was the SM algorithm which was first introduced by George Dantzig in late 1940s. The introduction of the SM happened simultaneously with the realisation of LP as an effective modelling technique for practical decision makings. To select the efficient method to solve LP problem, various criteria are considered such as size, sparsity, structure and the problem dimension. For example, for LP problems with high number of dimensions, the IPM is more effective than SM. In case of hyper-spare LP problems SM is virtually unbeatable (Gondzio, 2016).

The SM exploits the insights provided by the fundamental theorem of LP which states that the optimal solution if it exists is at one of the vertices of the feasible poly tope. SM gets a solution of a LP problem by visiting a sequence of vertices of the polyhedron, moving from each subsequent vertex to an adjacent one characterised by a better objective function value along edges of the feasible region until it reaches to termination vertex. Thus, in this scenario all the vertices of the feasible polyhedron must be visited before an optimal solution is reached (Ekwonwune & Edebatu, 2016).

On the other hand, affine IPM is a computation mathematical method that is used to solve LP problems in a very effective, reliable and accurate manner. In fact, it has won the interest of various operational and engineering researchers especially since their reintroduction by Karmarkar in (1984). The main concept of IPMs is basically different to the one that inspires the SM when solving LP problems. IPM reaches an optimal solution by moving through the interior of the feasible region while staying away from the boundary and staying closer to the central path (a smooth curve in the interior feasible region which serves as a guide to the set of optimal solutions). This is done by creating a family of parameterised approximate solutions that asymptotically converge to the exact solution (Boah & Twum, 2017; Hager & Zhang, 2014; Pan, 2013).

1.3 Company Background

LANA Company for Food Products Ltd. is a leading food canning company in Saudi Arabia established in the year 1995. Having an annual capacity of 55 million cans we produce various types of canned beans and peas.

Specializing in Legumes we offer a wide range of products like (Broad Beans), Foul with Chick Peas, Red Kidney Beans, White Beans, Chick Peas and Green Peas.

Brand name "WEDYAN" is well known in Saudi Arabia, Middle East and European countries. Exported to some of the countries like Germany, Italy, Sweden, Denmark, Australia, UAE, Kuwait, Oman, Qatar, Lebanon, Jordan, Syria, Yemen, Libya, etc.

1.4 Problem Background

This study aims to develop LP mathematical constrained model to maximize the profit of LANA Company for Food Ltd. In order to solve the developed LP model two mathematical optimization models are developed which are SM and affine IPM. In addition, the study aims to compare the profit obtained from the developed models with the industry's actual profit. This study is crucial as it allows the industry to employ some constraints such as the number of products and cost to examine the budgetary planning to obtain maximum profit.

1.5 Research Problem Statements

Recently, various methods have been developed to solve LP problems. However, there is always considerable demand to determine the efficient method to solve large scale LP problems particularly in applied science, engineering and economics. Moreover, the exhausted sophisticated codes which are required to solve linear programming models are quite expensive, time consuming and tedious. Thus, alternative methods are needed to solve large scale linear programming problems such as SM and IPM.

Besides that, in large manufacturing industries various drawbacks arise particularly those related to maximizing profit, minimizing cost, planning and allocating scare resources, etc. Therefore, the industries are in need for a mathematical modelling system that can be utilized specifically to improve the budgetary planning of the profit. Because current real systems which relies on trial-and-error concept are very expensive, slow and possess high risks.

Among the problem statements are the followings:

i. The necessity to examine budgetary process of the LANA Company for Food ltd Jeddah, Saudi Arabia which is significant to meet the expectation and demands of the management and to maximize the profit.

ii. Limited numbers of pre-determined products are produced by LANA Company each month which create significant demands to certain products to be maximized.

iii. The inability of the industry to successfully integrate budgetary manufacturing process with all production stages which creates defects on some products and contribute to low profit and productivity.

1.6 Research Objectives

The main objective of this research are the followings:

i. To formulate a Linear Programming (LP) model to maximize profit for LANA Company.

ii. To employ Simplex Algorithm to solve linear programming for maximizing profit for LANA Company.

iii. To develop the Interior Point Method to solve the linear programming model for maximizing profit of LANA Company.

iv. To compare the obtained profit from the developed models (SM and IPM) with the LP.

1.7 Research Scope

This study emphasizes the use of Linear Programming (LP), Simplex Method (SM) and Interior Point Method (IPM) to maximize the profit of LANA Company. Thus, the scope of this study is limited to these methods and to the data obtained from LANA Company.

1.8 Research Significance

The primary purpose of this study is to optimize the budgetary planning process of LANA Company. The significances of this study to the industry can be summarized as to maximize the profit of production process in the factory through the use of mathematical constrained optimization models including LP, SM and affine IPM. By developing LP, SM and affine IPM, the budgetary planning and resources allocation will be carried out efficiently.

Besides that, this study will provide insights and a deep understanding of the applicability of LP models in industry and how to formulate SM and affine IPM for solving real world linear problems. Moreover, it will present a better way to deal with decision making problems through the development and comparison of the SM and affine IPM to solve LP optimization problem to maximize profit. Finally, to other researchers particularly of similar interests who are undertaking further investigation on this topic, this study can be vital as a secondary source of information and guidance.

1.9 Research Organization

This thesis comprises of five chapters which are organized as follow:

Chapter 1 is devoted to bring about the research background on the mathematical constrained optimization models and their importance for solving various problems in engineering and applied science. It also addresses the Linear Programming (LP), Simplex Method (SM) and Interior Point Methods (IPM)s. In addition, this chapter presents the statements of problem, objectives, scope and significance of this research. It also addresses the overall research flowchart and finally ended up with the thesis organization and summary of the chapter.

Chapter 2 is dedicated to demonstrates in depth the relative literature on LP, SM and IPMs for solving problems related to maximizing profit in industries. This chapter is essential since it serves as the first step towards understanding the concept of LP constrained mathematical model, SM and IPMs. It contains the background, LP, SM, IPMs and finally the summary of the chapter.

Chapter 3 is the methodology chapter which is essentially elaborating the methods employed to obtain the optimal profit. It starts with the introduction, methodology flowchart, data acquisition, followed by the LP constrained mathematical model, SM and IPM models ends up with a summary of the chapter.

Chapter 4 is the result and discussion chapter which is devoted to illustrate the findings and discussions of this study. It contains introduction, results of LP, SM and IPM as well as the results of their comparison with the LP. It also presents the sensitivity analysis results of the LP, SM and IPM. In all presented results, a related discussion is presented to further elaborate the results.

Chapter 5 is the last chapter which is dedicated to provide a brief conclusion and remarks of the study, including introduction, recapitulation of research objectives. The majority of this chapter however, devoted to conclude the findings and elaborate the implications and limitations of the study.

1.10 Summary of Chapter 1

This chapter brought about the essential fundamentals and general introductory about the constrained mathematical models particularly the LP, SM and IPM. More specifically, it addressed the importance of IPM and SM to solve the LP problems in industries and various applied science applications such as the maximizing of profit. It also presented the problem statements, objectives, scope, significance of this research and ended up with the organization of the dissertation.

CHAPTER 2 : LITERATURE REVIEW

2.1 Introduction

In the previous chapter, the general introductory on the mathematical constrained models; Linear Programming (LP), Simplex Method (SM) and Interior Point Method (IPM) was exposed. Meanwhile, this chapter attempts to demonstrate in-depth relevant literatures on LP, SM and IPM for solving linear problems particularly in industries to maximize profit and minimize cost. The literature serves as the initial step toward understanding the basic theoretical and mathematical concepts behind the developing LP, SM and IPM models. This chapter contains background, LP, SM, IPM and the summary of the chapter. Toward the end of this chapter, various significant related studies on the application of LP, SM, IPM for maximizing profit and minimizing cost have been reviewed and addressed accordingly.

2.2 Background

Over the past decades, various techniques have been implemented to improve budgetary planning in industries mainly to solve the maximization and minimization problems in the presence of specific controlling parameters or constraints that determine the best solution. Indeed, optimizing and utilizing resources for budgetary planning has gained wide consideration and interests of many researchers particularly to maximize profit and minimize production cost in industries (Sahinidis, 2004).

Nowadays, profit maximization has become a major concern of almost all organizations and companies particularly in profit seeking establishments. According to Anieting, Ezugwu and Ologun (2013), managers seek to maximize profit by tackling the followings; competitors, capital funding, reducing the cost of operations and ultimately profit maximization. In an attempt to solve these problems, two operational techniques can be applied including the qualitative and the quantitative. The quantitative technique involves representing the real-world problems into mathematical forms to ultimately aid in the decision-making process. One of the most common quantitative techniques for allocating resources to products manner including profit maximization and cost minimization is Linear Programming (LP) (Yahya, 2004).

In today's competitive markets, companies urge to produce high quality products at the lowest possible cost while achieving maximum profit and meeting customer demands. In fact, a company's endurance in a competitive market strictly relies on its ability to manufacture high quality products at lower costs. Based on this, Ezema and Amakom (2012) emphasized that, organizations in the world are challenged by shortages of production inputs and low capacity utilization that can consequently lead to low production outputs and costs. Thus, industrial companies and organizations have to invent a management systems to guide and control their performance in processing and allocating resources to maintain quality and profitability.

Ahn and Haugh (2015) addressed the effectiveness of LP as a mathematical optimization tool for determining an optimal solution from a set of an alternative solutions with respect to an evaluation criterion (objective function) and linear mathematical constraints (inequalities or equalities). In this regard, LP problems can be solved with various methods for example Simplex Method (SM) and Interior Point Methods (IPM)s.

Although there are various methods to solve the LP problem. The focus of this literature is limited to the SM and IPM methods. To support this, Tesfaye, Berhane, Zenebe and Asmelash (2016) claimed that profit is factually influenced by the cost of resources and resource utilization.

2.3 Linear Programming

LP was introduced as an efficient technique to solve linear problems by Dantzig in 1947 and since then it has been implemented for various practical and industrial applications (Sulemana & Haadi, 2014). Dantzig defined LP as an extremely revolutionary tool that able to formulate real-world problems in detailed mathematical modelling, to state goals and to select the best decision when facing with a practical situation of great complexity (Dantzig, 1949).

Nowadays, LP is being successfully applied to solve linear decision-making problems including capital budgeting, conservation of resources, personnel allocation, promotion planning and advertising, design of diets, games of strategy, transportation systems and economic growth prediction (Adedayo, Ojo, & Obamiro, 2006; Anderson, Sweeney, Williams, Camm, & Cochran, 2015; Chaharsooghi & Jafari, 2007; Mehdipoor, Sadr-ol-ashraafi, & Karbaasi, 2006; Nahvi, Daghighi, Nahvi, & Kim, 2017; Popoola, Susu, Lateef, & Grema 2015). Generally, LP deals with a sort of programming problems where the objective functions and the relations between the variables and the resources are linear. Any LP comprises of four parts: The parameters, a set of decision variables, a set of constraints and the objective function (Anieting et al., 2013). Moreover, since their introduction, LP have gained wide consideration from an operational, engineers and applied sciences researchers (Maurya et al., 2015). The reasons behind this expansion is the vast applicability of LP, the widespread availability of LP software packages and their relative ease in determining the best solution.

The LP decision variables are the linear physical quantities with any set of possible values controlled by the decision maker and represented by mathematical symbols. The decision variables must be continuous; they can take on any value within some restricted range (assumptions requirements). The objective function (evaluation criterion) is basically a linear mathematical function of the decision variables which defines the criterion to evaluate and measure the optimal solution for the LP problem. It can also be used to specify the direction of the optimization either to minimize or maximize. The constraints of LP model are a set of linear mathematical functional equalities or inequalities that represent physical, technological, economic, ethical, legal or other restrictions on what numerical values can be assigned to the decision variables (Akpan & Iwok, 2016; Anieting et al., 2013).

LP model aims is to obtain optimal values for the decision variables that minimize or maximize the objective function and to satisfy all the constraints (Lord et al., 2013a). Thus, LP is well acknowledged as one of the most popular and is a mathematical model used to determine the best allocations of resource firm, which are limited (Anderson et al., 2015; Woubante, 2017). The procedure of developing LP is described in Ekwonwune and Edebatu, (2016) and summarized in the following sentences:

i. To determine the problem type in order to specify the type of the optimization problem either to maximize or to minimize certain function and then developing a mathematical expression for that particular problem called objective function.

ii. To identify all of restrictions, requirements and limitations (constraints) and expresses them in mathematical expression.

iii. To determine the linearity of the problem as mathematical expression to better describe the systems based on objective function as well as the constraints.

iv. To Identify the resources and decision variables with respect to the objective function.

v. To ensure all solved variables are non-negative. LP requires the solved variables to be positive, equal or more than zero.

In industries, there are different optimization problems such as to maximize or minimize economic function or a ratio of physical function including cost/volume, cost/time, profit/cost and cost/benefit. Thus, there is a need for developing optimization model to solve these types of problems. Such model is known as the LP model which is expressed mathematically (Saha, Hossain, Uddin, & Mondal, 2015) as follows:

$$Minimize(orMaximize)Z = \sum_{j=1}^{n} c_j x_j \qquad (2.1)$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \{\leq, =, \geq\} b_j \,;\, i = 1,2,..,m \qquad (2.2)$$

$$x_j \geq 0; b \geq 0$$