Approximating One-Dimensional Coupled Shallow Water Equation for Predicting Tsunami Wave Propagation Using New Semi-Implicit Method

by

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A dissertation submitted in partial fulfillment of the requirements for the degree
of Master of Science (Engineering Mathematics)

Institute of Engineering Mathematics

UNIVERSITI MALAYSIA PERLIS

2016

ACKNOWLEDGEMENT

First of all, I would like to express my gratitude to Allah the Almighty, who bestowed me the strength, the knowledge, and the devotion to complete this dissertation.

I would like to express my gratitude and thanks to my supervisor Dr. Nursalasawati bt Rusli for her professional guidance, continuous support, her precious time, and her friendly attitude towards me to guide me to reach this level.

I sincerely thank my parents Mohd Kanafiah Salleh and Azizah Awang Ngah and also thanks to my siblings, especially Siti Nurul Aqmariah for their support, encouragement and patience they have given to me throughout the duration of running this dissertation.

I would also like to thank my friends, especially Norbazilah, Wan Nor Alia, Noramirah, Intan Mastura and Farhah Athirah for their opinions, advices and help throughout the process of completing the dissertation. Their supports mean a lot to me in achieving the objectives of this dissertation.

I want to express my appreciation to my external examiner, Professor Dr. Zurni Bin Omar on his willingness to check my dissertation, his opinions and advices throughout the process of completing the dissertation.

Finally, I would like to convey my appreciation to all persons involved both directly and indirectly for their contribution and support to the success of my dissertation. The thoughtfulness and kind help are much appreciated.

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LIST OF SYMBOLS

c	Courant number, $\Delta t / \Delta x$
D	Total of h and η
h	Water depth
g	Gravitational acceleration
η	The height of water wave
M	Product of depth average velocity and water velocity
Δt	Time step size
Δx	Spatial step size

LIST OF ABBREVIATION

1D	One Dimensional
2D	Two Dimensional
3D	Three Dimensional
CFL	Courant-Friedrichs-Lewy
FDM	Finite Difference Method
FEM	Finite Element Method
FVM	Finite Volume Method
SWE	Shallow Water Equation
WWP	Water Wave Propagation

Penghampiran Pasangan Persamaan Air Cetek Dalam Satu Dimensi untuk Meramalkan Perambatan Gelombang Tsunami Menggunakan Kaedah Separa-tersirat Yang Baru

ABSTRAK

Tsunami adalah bencana alam yang jarang berlaku jika dibandingkan dengan bencana alam yang lain. Pertumbuhan penduduk di kawasan pesisir pantai akan meningkatkan statistik bilangan mangsa yang terjejas teruk akibat bencana tersebut. Dalam usaha untuk menyiapkan diri menghadapi bencana tsunami, ramalan tentang gelombang tsunami perlu dimodelkan. Kajian ini membentangkan pasangan persamaan air cetek dalam satu dimensi bagi perambatan gelombang tsunami. Simulasi pemodelan berangka perlu dibangunkan untuk meramalkan kejadian perambatan gelombang tsunami. Untuk menyelesaikan persamaan air yang cetek, kaedah separa tersirat telah digunakan dalam proses pendiskretan. Gelombang permukaan bebas redaman telah digunakan dalam mengesahsahihkan kaedah separa tersirat. Selepas proses pengesahsahihan, algoritma berangka dilaksanakan menggunakan perisian MATLAB untuk menyelesaikan pasangan persamaan air cetek dalam satu dimensi bagi perambatan gelombang tsunami. Bagi tujuan simulasi, syarat sempadan dan syarat awal, langkah ruang dan langkah masa serta penghampiran persamaan air cetek telah dikodkan. Hasil simulasi persamaan air cetek dalam satu dimensi berjaya meramalkan keadaan perambatan gelombang tsunami berhampiran dengan pantai selepas gempa bumi di dasar laut berlaku.

Approximating One-Dimensional Coupled Shallow Water Equation for Predicting Tsunami Wave Propagation Using New Semi-Implicit Method

ABSTRACT

Tsunamis are uncommon events compared to natural hazards. Population growth along the shorelines has increased the statistic of the number of victims affected by the tsunami disaster. In order to help people to be aware about the tsunami disasters, a tsunami wave propagation must be predicted. This study proposed a one dimensional coupled shallow water equation for predicting the tsunami wave propagation. The simulation of numerical modelling was conducted to predict the occurrence of tsunami wave propagation. In order to approximate the shallow water equation, the discretization process was conducted using a new semi-implicit method which is one of the methods in finite difference method. A problem regarding the free surface wave damping problem were used in validating the semi-implicit method. After the validation process, the numerical algorithm was developed in MATLAB software to solve the one dimensional coupled shallow water equation for the tsunami wave propagation. For the purposes of the simulation, the boundary condition and initial condition, the spatial steps and time steps as well as the approximation of shallow water equation were encoded. The simulation results of one dimensional shallow water equation successfully predicted the behaviour of the tsunami propagation near the coastline after the earthquake ground shaking occurred.

CHAPTER 1

INTRODUCTION

1.1 Overview

The waves occurring in the oceans and along the shores can be classified based on the period, wavelength or the nature of the force that generates free surface disturbance (Benoit, 2014). From the hydraulic point of view, water wave propagation (WWP) is one of the waves occurs in the oceans, which can be categorized into many types.

Tidal wave and tsunami wave are the examples of WWP occurred in the oceans. Tidal waves are the waves that are generated by the gravitational forces of the sun, the moon and the earth which often occur in the coastal regions. In contrast, the tsunami wave propagation is formed by the movement of sea bed due to the earthquakes (Goto et al., 1997). UNESCO (2014) stated that the earthquakes take place underneath or near the ocean would only cause the tsunami waves. Tsunami belongs to long waves that are propagated in a very high speed with its wavelength longer than the depth of water (Imamura et al., 2006). This scenario was described by Imamura et al. (2006) as characterized by the shallow water equation (SWE).

SWE has been used in many applications regarding the water flow, including the tsunami propagation, storm surges, solute transport, river flow and inundation, as well as ecological model (Jakeman, 2006). The behaviour of the SWE as defined earlier truly describes the WWP (Liu et al., 2009). Besides, the natural condition of the shallow water when the waves approaching the coastal area and entering the shallow area depicted the characteristic of the shallow water such that the WWP descreases in speed when entering the shallow area. Various numerical schemes have been used in solving one dimensional SWE such as finite difference method (FDM) (Causon & Mingham, 2010), finite element method (FEM) (Comblen et al., 2009) and finite volume method (FVM) (Castro et al., 2008).

1.2 Research Background

In this research, several information regarding the tsunami wave propagation problems is being used as the backbones of the research. The description about the SWE and its behaviour are discussed in this section.

1.2.1 Water Wave Propagation

Wave propagation is a physical phenomena that occurs in our daily life. Ordinarily, wave propagation takes place in many conditions that involve air, water and also acoustic wave. For example, one of the natural disasters that happen recently in Malaysia around the coastal area in Kelantan is the WWP. On 6th of February 2016, the WWP or known as tidal wave had destroyed many houses near the coastal area in Kota Bharu (Muhammad, 2016). A strong wind with a speed around 50 to 60 km/hour from the east coast passed and simultaneously generated a tidal wave in the shallow water area or coastal area. This phenomenon had occured because of the gravitational forces of the sun, moon and earth. Other than that, the tsunami propagation can also be categorized as for WWP problem. The most recent example was the tsunami wave propagation occurring in Chile, Latin America. According to UNESCO (2015) the earthquake with a magnitude of 8.3 took place in the seabed and caused a tsunami wave in Chile in September 2015. Under those circumstances, the tsunami disaster had caused 156 deaths. Based on the fact by Muhammad (2016) and UNESCO (2015), the tsunami propagation gives much more impact compared to the tidal waves. Therefore, in order to predict the WWP, this study will focus on tsunami wave propagation.

1.2.2 Tsunami propagation

Based on the reviews from Shuto and Fujima (2009), it can be concluded that the tsunami wave propagation transpired from the earthquake shaking ground in the ocean. Most of the tsunami occurs in the Pacific Ocean, but it can also threaten the coastlines of several countries in the Indian Ocean, the Mediterranean Sea, the Caribbean region and the Atlantic Ocean (UNESCO, 2014).

One of the worst cases that involves the tsunami wave propagation occurred was in the Indian Ocean. In fact, in 2004, the devastating cause tsunami’s attack the countries around the Indian Ocean was due to the lack of tsunami warning system around there. Thousands of lives were lost during the horrendous tsunami disaster (Teh et al., 2009). In that case, many significant improvements have been made in earthquake and tsunami sciences in order to reduce the disaster risk (Satake, 2014). Imamura et al. (2006) stated that tsunami wave belongs to a shallow water wave is desirable to describe the tsunami wave propagation. After the global surveys and mapping which proved that the ocean’s depth or bathymetry data can be used in a simulation of the tsunami wave propagation because it was shown that the wavelength of water wave is much longer than the depth of water. For this reason, this study will be conducted in order to predict the initial tsunami wave propagation after the earthquakes in the seabed with the initial and boundary condition that were provided by Park (2007).

1.2.3 Shallow Water Equation

In the past few years, the shallow water model has been used in modelling the physical phenomena of the water flows such as the flood waves, dam-breaks, and bore wave propagation in the river (Delis & Katsaounis, 2005). As we know, the equations that involved in fluid flow problem were often classified as hyperbolic equations. In this study, the SWE was used to model the tsunami wave propagation. The following Equation (1.1) is the Navier-Stokes equation that is derived by following the conservation of mass, momentum and energy (Hoffman & Chiang, 2000).

$$\frac{\partial Q}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}+\frac{\partial G}{\partial z}=\frac{\partial E_v}{\partial x}+\frac{\partial F_v}{\partial y}+\frac{\partial G_v}{\partial z} \tag{1.1}$$

where Q is the continuity variable with respect to time, t. E, F and G are the spatial component in x, y and z respectively and v represents the right hand side expression of E, F and G.

Furthermore, the Navier-Stokes equations are the one of the complex equations that is widely used in solving the fluid mechanics problem. Mungkasi (2014) stated that the Navier-Stokes equations were used to simulate the compressible and incompressible fluid flows. From the above fact, the SWE are arising from the fundamental equation in fluid mechanics problems (Jakeman, 2006).

SWE is important for a variety of problems in the coastal and environmental engineering such as for the tidal flow, transient wave phenomena and for the transient pollutant transport. The fact that ruinous tsunami in the Indian Ocean in 2004 which involving hundred kilometers long of waves but the depth of water is about two or three kilometers described the trouble for the coastal and environmental engineering (Moler, 2013). In this situation, the shallow water approximation provides a reasonable model which is the model of water flow that can be used in many problems for simulating the problem regarding the WWP (Causon & Mingham, 2010). Jakeman (2006) stated that the SWE was used to represent the water flow in the horizontal axis in the water body that the wavelength is greatly elongates more than the depth of water. In the mathematical model, SWE is important in investigating the wave flow problem in the oceans that was stated by Wesseling (2001) where it specifically described the behaviour of WWP. For example, the propagation of tsunamis described the WWP based on shallow water theory very well. Liu et al. (2009) came out with a simple practical model which described the propagation of tsunamis. Therefore, the SWE is needed in order to predict the tsunami wave propagation.

As a result, the height of the wave and the velocity of WWP must be derived from SWE to give the simulation result which mimic the real situation of tsunami wave propagation (Crowhurst, 2013). After some derivation, Equation (1.1) is reduced to form the SWE in dimensional form. The following formulations are the three dimensional (3D) SWE which were reduced from the Navier-Stokes equation.

$$\frac{\partial \eta}{\partial t}+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1.2}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}\right)=0 \tag{1.3}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{1}{\rho}\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}\right)=0 \tag{1.4}$$

$$g+\frac{1}{\rho}\frac{\partial \rho}{\partial z}=0 \tag{1.5}$$

where x and y are horizontal axes, z is the vertical axis, t for the time, h the water depth, η the vertical displacement of the water surface above the still water surface, u, v and w are water particle velocities in the x, y and z directions, g is the gravitational acceleration, τ is the normal or tangential shear stress, p is the hydrostatic pressure and ρ is the density of the water.

Then, the 3D equation in Equations (1.2) - (1.5) will be reduced to two dimensional (2D) SWE as shown in Equation (1.6) – (1.8) which is known as the shallow water theory.

$$\frac{\partial \eta}{\partial t}+\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}=0 \tag{1.6}$$

$$\frac{\partial M}{\partial t}+\frac{\partial}{\partial x}\left(\frac{M^2}{D}\right)+\frac{\partial}{\partial y}\left(\frac{MN}{D}\right)+gD\frac{\partial \eta}{\partial x}+\frac{\tau_x}{\rho}=A\left(\frac{\partial^2 M}{\partial x^2}+\frac{\partial^2 M}{\partial y^2}\right) \tag{1.7}$$

$$\frac{\partial N}{\partial t}+\frac{\partial}{\partial x}\left(\frac{MN}{D}\right)+\frac{\partial}{\partial y}\left(\frac{N^2}{D}\right)+gD\frac{\partial \eta}{\partial y}+\frac{\tau_y}{\rho}=A\left(\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial y^2}\right) \tag{1.8}$$

where D is the total water depth given by $h+\eta$, while τ_x and τ_y are the bottom frictions in the x- directions and y-directions, respectively. A is the horizontal eddy viscosity which is assumed to be constant in space. M and N are the discharge fluxes in the x- and y- directions.

Now, the 2D SWE in Equations (1.6) until (1.8) were reduced to one dimensional (1D) SWE. The 1D SWE is given in a coupled equation which is,

$$\frac{\partial \eta}{\partial t}+\frac{\partial M}{\partial x}=0 \tag{1.9}$$

$$\frac{\partial M}{\partial t}+gD\frac{\partial \eta}{\partial x}=0 \tag{1.10}$$

where D is the total water depth at x , and M is a quantity defined as product of depth averaged velocity and water velocity in the $+x$ direction and η is the height of water wave from the water surface. In order to simulate a simple model for prediction for tsunami wave propagation, the 1D SWE without bottom friction were used for the first step. Therefore, the 1D SWE will be used in this study based on the previous justification. Figure 1.1 shows the water level profile for 1D SWE.

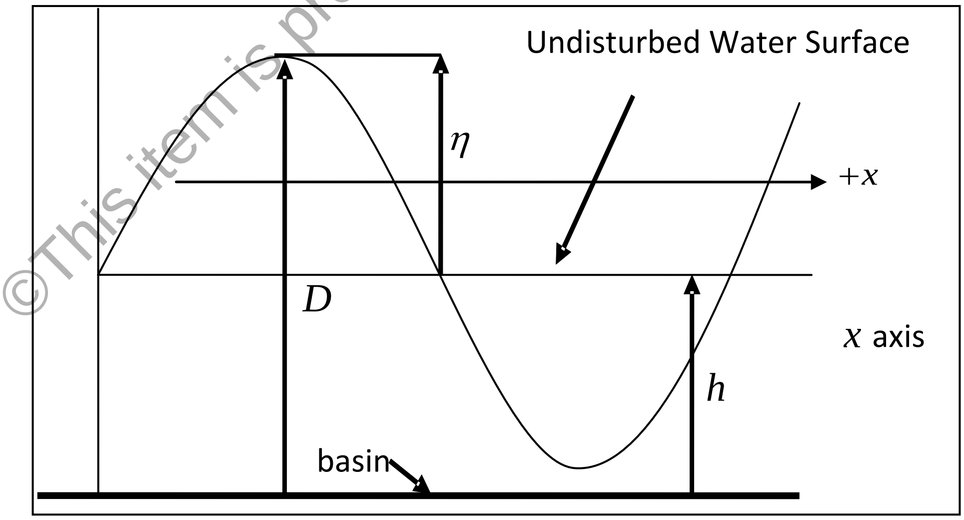

Figure 1.1: Water level profile for one dimensional shallow water equation.

Figure 1.1 describes the terms that were used in 1D SWE. From the illustration in Figure 1.1, the height of water wave, η and the product of depth average velocity and water velocity in $+x$ direction, M will be implemented in Matlab for predicting the tsunami wave propagation.

1.3 Problem Statement

Tsunamis are uncommon events compared to other natural disasters. However, population growth along shorelines has increased in number and the impact of tsunami disasters, as tsunamis can rapidly develop and grow in shallow coastal water. Based on the fact stated in section 1.2.2, this study is conducted in order to predict the behaviour of the tsunami wave propagation. Significantly, the deportment of the SWE delineates the tsunami waves where the wavelength of WWP is horizontally longer than the depth of water. The 1D coupled SWE will be used which was explained in Section 1.2.3 in order to clarify how to predict the tsunami wave propagation. The new semi-implicit method will be used in order to solve the 1D coupled SWE. Later, by applying the numerical method, the simulation of tsunami wave propagation will be implemented and the tsunami wave can be foretold to the people of possible incoming disaster.

1.4 Objectives of the Study

The main objective of this study is to predict the behaviour of tsunami wave propagation. In order to achieve the main objective, the following sub-objectives will be carried out:

(a) To discretize 1D coupled SWE using semi-implicit method.

(b) To develop a numerical algorithm and procedure for solving 1D SWE.

(c) To validate the numerical algorithm with θ-method.

(d) To verify the develop method in tsunami propagation.

1.5 Scope of the Study

In this study, the work is focused on solving the non-linear 1D coupled SWE. One of the numerical techniques which is the semi-implicit method was selected to solve the SWE. The discretization domain is set about 100 meters towards the coastline or in the $+x$ direction in the grid. In addition, the arrival time for prediction of tsunami propagation to the coastline is set to 14 seconds based on the formula given by Liu et al. (1995). As an illustration, the discretization of SWE and the value set up for time, t and direction, x will be compiled in MATLAB to provide the approximation solution and predicting the tsunami waves after the earthquake ground shaking occur.

1.6 Significant of the Study

In order to prevent the lost of many lives, a mathematical model is needed. Therefore, it is important to predict the propagation of tsunami’s wave occuring near the shores. For the case when the tsunami’s wave propagation happens, the numerical modelling of simulation of tsunami wave propagation will be developed. From the study, a more realistic mathematical models can be developed in order to interpret and enhance the understanding of the characteristics of the tsunami propagation through mathematical behaviour. The advancement of computer knowledge is significant for this study to solve the complex mathematical problem. Therefore, the numerical algorithm that has been done in this study can be enhanced in 3D SWE.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter reviews the existing research on tsunami wave propagation and the method used in solving the SWE. In the first place, the exploration about how the waves come about and the advantage of predicting the tsunami wave is discussed. In the followings, the existing equation and method for predicting tsunami wave propagation will be elaborated. Finally, the deliberation will be on the new semi-implicit method for SWE in prognosticating the tsunami wave propagation.

2.2 Impact of Tsunami

Coastal areas are extremely susceptible to natural hazards associated with the sea. Indeed, the disaster in the sea caused the tsunami wave to occur and achingly damaged the coastline area. Periáñez and Abril (2013) developed a model for tsunami wave propagation in South of Iberia and tested the historical tsunami, including the 1755 Lisbon tsunami and the Algeria tsunami in 1875 and 2003. The model then has been applied to simulate the tsunami propagation in the former Gulf of Tartessos. The results indicated that a tsunami could not damage the Gulf's interior because of the reduction of wave height and water velocity when entering the Gulf. Notwithstanding, the populations around the Gulf's boundary have undergone severe flooding. Based on the fact earlier, the tsunami propagation would only damage the area of the coastline. Teh et al. (2009) developed the analytical and numerical model to assess the role of mangroves in mitigating the tsunami waves around the Penang Island and the northwest of Peninsular Malaysia. The results denoted a significant ratio reduction in the height and velocity of the wave with the existence of the mangrove forest. Therefore, tsunami propagation would be reduced its height and velocity if the wave arrived and enter the mangrove forest area.

However, the tsunami propagation still cannot be avoided from damaging and harming the coastline. One reason is because over the time, the population has increased along the shorelines so thus the impact of tsunami disasters (Flouri et al., 2013). In order to avoid any possible disasters, Rabinovich and Eble (2015) made some overview about the measurement of the deep-Ocean tsunami for predicting the tsunami wave generation from 50 years ago to the present day. Measurement of the deep-ocean plays a major role in understanding the physics of tsunami wave propagation and enhances the effectiveness of tsunami warning system. They reported that the deployment and the maintenance of the instrument in the deep - ocean system are required on the ground area in order to transmit the data in real time; hence resolving the tsunami waves in the open ocean. This measuring practice shows that the tsunami wave propagation can be predicted earlier if the deep-ocean is measured using some instrument that has been deployed in the ocean and therefore reduced the number of lives lost.

A significant effort has been made in implementing tsunami simulation models and in supporting the tsunami hazard’s assessment on coastal areas worldwide. Flouri et al. (2013) developed a simulation to calculate tsunami inundation from four different earthquake scenarios in Heraklion, Greece. From the review, the wave height and the arrival time of the tsunami wave from 5 gauges that were placed offshore in the city of

Heroklion were calculated. In order to show the mitigation of tsunami hazards, the numerical results were projected in digital map which are useful in calculating the population, structures and facilities endangered by the tsunami. The results indicated that, it is important to introduce metrics that characterize the tsunami's impact in order to identify zones of highly potential from any tsunami disaster (Flouri et al., 2013).

Moreover, the study of the effect of grid size and time step in simulating the tsunami propagation was studied by Kian et al. (2014) using the non-linear SWE. They employed two modelling programs which are NAMI DANCE and FUNWAVE where several grid sizes and time steps were selected and have been tested in the simulations. The results showed that the tsunami leads to greater wave height when the grid size selected is smaller than the water wave height. Therefore, it can be concluded that the study of the effect on grid size and time step may affect the behaviour of the amplitude of water wave during the simulation process (Kian et al., 2014).

2.3 Existing Methods for Predicting Tsunami Wave Propagation

Many tools and methods using numerical modeling were used for assessing coastal vulnerability to the tsunami disaster problem. From the readings, most methods that were introduced have been using the leap-frog scheme for solving SWE such as Imamura et al. (1988) who used a finite difference model for simulation of transoceanic tsunami. Several years later, the model has been improved by Cho and Yoon (1998) to achieve proper dispersive effects for tsunami propagation. Even so, the numerical models are still limited to the case of constant water depth. Thus, Cho et al. (2007) concluded that both numerical models have limitations whereby the size of spatial and time step should be changed continuously when the water depth changes.