An Extended Stochastic Goal Mixed Integer Programming Approach for Optimal Portfolio Selection in the Amman Stock Exchange

by

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LIST OF ABBREVIATIONS

AFM	Amman Financial Market
ALM	Asset Liability Model
APM	Asset Pricing Model
APOT	Arab Potash Co.
ARAS	Arab Assurers
ARBK	Arab Bank
ASE	Amman Stock Exchange
CAPM	Capital Assets Pricing Model
CCP	Chance Constrained Programming
CEO	Chief Executive Officer
CPU	Central Processing Unite
CRRA	Constant Relative Risk Aversion
CV	Coefficient Variance
CVaR	Conditional Value at Risk
DJIA	Dow Jones Industrial Average
DP	Dynamic Programming Portfolio
ESGMIP	Extended Stochastic Goal Mixed Integer Programming
FTSE	Financial Times Stock Exchange - London
GP	Goal Programming
GBM	Geometric Brownian Motion
GMD	Gini's Mean Difference
IFC	International Finance Corporation of World Bank
ILP	Integer Linear Programming
JPHT	Jordan Phosphate Co.
JSC	Jordan Securities Commission
JCB	Jordan Central Bank
JTEL	Jordan Telecom
LQC	Linear Quadratic Control problem (projected policy)
MAD	Mean Absolute Deviation
MCDM	Multiple-Criteria Decision Making
MEDI	Middle East Div

MOCCP	Multi Objective Chance Constrained Programming model
MP	Mathematical Programming
MPT	Modern Portfolio Theory
MV	Mean Variance
MVO	Mean Variance Optimization model
NAQL	Transport Barter
OPL	Optimization Programming Language
PSP	Portfolio Selection Problem
QMIP	Quadratic Mix-Integer Programming
QP	Quadratic Programming
RJAL	Royal Jordanian Airline
S&P	Standard and Pour
SA	Simulated Annealing metaheuristic
SDC	Securities Deposit Center
SGDEP	Stochastic Goal Deterministic Equivalent Program
SGP	Stochastic Goal Program
SLP	Stochastic Linear Programming
SLR	Simple Linear Regression
SGMIP	Stochastic Goal Mixed Integer Programming
SMIP	Stochastic Mixed Integer Programming
SP	Stochastic Programming
SPIC	Specialized Investing Co.
THBK	The Housing Bank
TSX	Toronto Stock Exchange
TWAP	Time-Weighted Average Price policy (deterministic)
VaR	Value at Risk
VIF	Variance Inflation Factor

LIST OF SYMBOLS

x_i The fraction of the portfolio invested in security *i* that is purchased in the first-stage (t=0).

n Total number of securities

y_{il}^{t} The fraction of the portfolio invested in security *i* that is purchased in the second-stage (t>0).

m Total of time period, for the simplicity $T = m$-1

L Total number of scenarios

$\emptyset_{il}^{t}$ The unit price of security *i* at time t = 0,1,…,m under scenario l = 1,2,… L.

z_{jl}^{t} Fraction of the portfolio invested in bond *j* to purchase at time *t* under scenario *l*.

h_j^* The time to maturity for each bond *j*.

φ_{jl}^{t} The price of bond *j* at time *t* under scenario *l*.

U_{jl}^{t} Bond return at maturity.

$\hat{B}$ Initial wealth of the portfolio.

p_l The probability of a scenario realisation

$\bar{\omega}_{il}^{0}$ Transaction cost at t=0

$\bar{\omega}_{il}^{1}$ Transaction cost at t=1

β^* Optimal beta

II^* Optimal information

g_i^0 The amount of stock *i* at t=0

δ_l^t Beta Penalty variable

ς^0 information Penalty variable

ξ_{sl}^{t} Sector Penalty

λ_l^t Liquidity Penalty

G^t The upper bound on the number of stocks to hold at time t

$G^{\sim t}$ The upper bound on the number of bonds to hold at time t

Pendekatan Pengaturcaraan Gol Integer Bercampur Stokastik Lanjutan untuk Pemilihan Potfolio Optimum di Bursa Saham Amman

ABSTRAK

Pemilihan portfolio optima telah mendapat perhatian besar terutama mengenai kekerapan ketidakstabilan di pasaran kewangan. Kajian ini dijalankan untuk menyiasat keberkesanan lanjutan *Stochastic Goal Mixed Integer Programming* (SGMIP) yang dilanjutkan untuk menyelesai dan memilih Portfolio dinamik optima yang dapat memuaskan citarasa pelabur di bawah persekitaran ketidakpastian, sebagai satu yang penting di dalam bidang kewangan dan pelaburan. Dua jenis Portfolio telah disiasat; Portfolio saham tulen dan Portfolio saham bon membentuk *Mixed Integer Programming* (MIP). Portfolio semasa melibatkan beberapa kekangan sebenar dunia seperti pelbagai jangkamasa, objektif (risiko, pulangan, maklumat, kecairan, kekangan kardinaliti, kepelbagaian, kostransaksi, dan kekal dalam prestasi pasaran) dan pelbagai asset telah membawa kepada masalah berskala besar. Penyelidikan ini bersifat kuantitatif, model pencampuran antara dua teknik pengaturcaraan, model GP sesuai untuk memenuhi pelbagai objektif dan rangka kerja SP bagi menawan beberapa ketidakpastian sumber digunakan untuk merumus masalah Portfolio. Semua100 syarikat yang tersenarai dan berterusan perniagaan di ASE telah dipilih sebagai sampel kajian ini, dan data itu dikumpulkan setiap hari untuk semua parameter saham individu. Pulangan harian Indeks Apongan ASE digunakan sebagai penanda aras Portfolio. Rumusan Brownian telah digunakan untuk meramal harga saham bagi masa depan. Hasil kajian dibentangkan secara komputasi, kewangan dan statistik. Penemuan kajian mengen engahkan fungsi penguraian a logoritma dalam meningkatkan peruntukan memori dan masa *Central Processing Unit* (CPU). Penyelesaian model lanjutan SGMIP mencapai pemilihan optima di dalam Portfolio saham tulen dan portfolio saham bon. Hasil yang menarik diperolehi daripada reka bentuk Portfolio a lgoritma, yang berada pada tahap pertama, hampir tidak ada perbezaan yang terdapat dalam prestasi antara Portfolio SGMIP dan indeks, setelah menambah maklumat (*drift*) objektif, Portfolio SGMIP mengatasi sampel dan pulangan portfolio Indeks. Portfolio saham bon SGMIP dilaburkan di dalam saham tulen pada peringkat pertama. Di dalam peringkat kedua, Portfolio melabur di dalam kedua-dua saham dan bon ketika berada di dalam senario yang terbaik dan stabil. Walau bagaimanapun, Portfolio bertukar pelaburan kepada bon apabila berada di dalam senario yang lebih teruk. Sebagai sebuah penyelidikan impirikal, set hipotesis penyelidikan impirikal telah diformulasikan dan Portfolio SGMIP telah diuji. Di bawah persekitaran yang tidak menentu, ketersediaan maklumat merasionalisasikan kepelbagaian apabila Portfolio dinamik melabur di dalam satu instrument kewangan (saham), dan cenderung kepelbagaian apabila melabur di dalam lebih daripada satu instrument kewangan (saham dan bon). Kerja ini membentangkan keaslian model lanjutan SGMIP untuk mencapai penyelesaian optimum. Sumbangan empirical kajian ini membentangkan model lanjutan SGMIP dengan menambahkan maklumat sebagai faktor baharu yang memilih elemen Portfolio, yang boleh dianggap sebagai sumbangan tersendiri di dalam ilmu pengetahuan. Reka bentuk penyelesaian a logoritma di dalam kajian ini dapat menyelesaikan masalah portfolio dengan menggunakan pulangan harian untuk meramalkan volatiliti dan parameter portfolio lain dan boleh dianggap sebagai sumbangan metodologi penyelidikan ini kerana hasil kajiannya mengatasi hasil kajian-kajian lain berdasarkan model pengaturcaraan yang sama, dan mengaplikasikannya di dalam dunia sebenar ASE.

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ABSTRACT

The selection of optimal portfolio received a great attention with regard to the frequent instability in the financial markets. This research was conducted to investigate the effectiveness of the ESGMIP in solving and selecting optimal dynamic portfolio that satisfying the investors' preferences under the uncertainty environment, as an important area in finance and investment. Two types of portfolios were investigated; a pure stock portfolio and a stock-bond portfolio formed a mixed Integer Programming (MIP). The current portfolios involved set of real-world constraints which are multiperiod, multiobjectives (risk, return, information, liquidity, cardinality constraints, diversity, transaction cost, and stay within market performance) and multi-assets resulted on a large- scale problem. This research is quantitative in its nature, an advanced model mixing between two programming techniques, GP model is suitable to satisfy multiple objectives and SP framework that captured numerous sources of uncertainty was used to formulate the portfolio problem. All listed and continuously traded companies (100) in ASE were selected as a sample of this study, and the data was collected on daily basis for all the parameter of individual stock. The daily return of ASE Float Index used as the portfolio benchmark. Brownian motion formula was used to predict the stock price in future time period. The results of the study were presented computationally, financially and statistically. Findings highlighted the role of the decomposition algorithm in improving the memory allocation and CPU time. Solving the extended SGMIP model reached the optimality in selecting both pure stock portfolio and stock-bond portfolio. A fascinating result was obtained from the portfolio algorithm design, which was in the first stage, almost no differences were found in the performance between the SGMIP and index portfolio, after adding the information (drift) objective, the SGMIP portfolio outperform the sample and Index portfolio return. The SGMIP stock-bond portfolio invested in pure stock in the first stage. In the second stage, when the situation is favourable to the investor such as in best and stable scenarios, the portfolio invested in both stock and bond. But, when the situation became bad, the portfolio switched to invest completly in bonds as in the worst scenario. As an empirical research, set of hypotheses were formulated and the resulted SGMIP portfolios were tested. Under uncertain environment, the availability of information rationalized the diversity when the dynamic portfolio invested in one financial instrument (stocks), and tend to be diversifiable when invested in more than one financial instrument (stock and bond). This work presents a novel extended SGMIP model to reach to an optimal solution. The empirical contributions of this research presented on extending the SGMIP model by adding information as a new factor that select the portfolio elements, which can be considered as a distinctive contribution to the knowledge. The decomposed algorithm solution that designed to solve portfolio problem and using a daily return to predict the volatility and other portfolio parameters considered as a methodological contribution for this research since its results outperform the results of the previous studies based on the same programming models, applying it on the real-world ASE.

CHAPTER 1

INTRODUCTION

1.1 Background to the Study

For decades, since 1926, financial market uncertainties and economic turbulence have resulted inglobal financial crises, which lead many corporations to bankruptcy. These crises induced the corporations and portfolio managers to become more careful investors. Financial institutions researched and enlisted portfolio management strategies to deal and overcome market uncertainties to achieve corporate goals. Financial innovations relating to the creation of new instruments by financial institutions and banks such as derivatives and other investment products, add a fair amount of complexity to the role of portfolio managers. Many researchers have tried to design safe portfolio models and investment strategies as an endeavour to outperform the repeated market failures, but commonly resulted in complexity in computation and challenges in finding solutions (Stoyan & Kwon, 2011).

An investment portfolio is a tool combined from two or more financial assets owned to maximise the market value and to achieve the optimal usage of these assets (Jones, 2004). The Optimal Portfolio Theory (OPT) was developed by Harry Markowitz in 1952. Markowitz optimized the selection of his portfolio by formulating the problem as Mean-Variance (MV) model in 1952, and as Quadratic Programming (QP) in 1987 (Ibrahim, 2008). The MV model helps investors to select a combination of assets based on the trade-off between risk and return. The combination of assets are chosen either by minimising the risk while achieving the pre-determined expected return, or maximising the expected return within the preferred level of risk (He & Qu, 2014).

Mathematical programming is used by several researchers to replace the QP model as the latter fails to take into account real-world constraints that are faced by investors while making decisions (Jobst, Horniman, Lucas, & Mitra, 2001; Ibrahim, 2008; Sawik, 2010; He & Qu, 2014). These constraints include the number of assets to be held (cardinality constraint), weight of individual asset, transaction size, transaction cost, liquidity, etc. (Bienstock, 1996; Mansini & Speranza, 1999). The presence of these constraints increases the complexity of the Portfolio Selection Problem (PSP) (He & Qu, 2014).

Another factor that affects an investor's decision-making is the uncertainty condition of the financial markets. However, the portfolio parameters of expected return and covariance are based on the historical data in MV and other models of PSP (Chang, Mead, Beasley, & Sharaiha, 2000; Kllerer, Mansini, & Speranza, 2000; Crama & Schyns, 2003; Mansini & Speranza, 2005; Konno & Yamamoto, 2005). Thus, an investor may employ a flexible strategy by rebalancing his portfolio periodically as the perception of asset price changes, regarding new uncertainties in the market (Yano, 2014).

Stochastic Programming (SP) is a common technique in portfolio decision-making to create a model for random uncertainty conditions in the financial markets. It deals with the uncertainties in a flexible way and incorporates real-world constraints easily (Yano, 2014). In order to investigate the dynamic portfolio, a formulation of Mixed-Integer Programming (MIP) could be applied with an objective and constraints that account for different financial factors and significant portfolio traits surrounding investment decisions. One of the main constraints of the model is the size of the portfolio, in which extensive studies were conducted by researchers such as Chang et al. (2000), Jobst et al. (2001), Crama and Schyns (2003), Yiu (2004), Shaw, Liu, and

Kopman (2008), and Stoyan (2009). The Stochastic-Goal Mixed-Integer Programming (SGMIP) model developed by Stoyan (2009) presents liquidity and transaction cost as additional constraints, besides portfolio size.

A study on the Jordanian stock exchange, Amman Stock Exchange (ASE), is performed to present all aspects of uncertainty and market fluctuation. From January 2006 up to January 2013, ASE faced a decrease in the Free Float Index. As illustrated in Table 1.1, the Index lost 2461.2 points, which represented a drop of more than fifty percent of its market value. In 2013, the market condition improved and continued on during the first quarter of 2014. Dabbas (2013) found many factors that contributed to the improved performance of ASE, including price movements, value increments of daily trades or the number of transactions, and prominently, stability in political and security status, as compared to the neighbouring countries at the time.

Table 1.1: ASE Free Float Index Price by Point.

Year		**Index**		**Change %**
		Opening price	**Closing price**	
2005		2837.7	4259.7	0.50
2006		**4419.7**	3013.7	(0.32)
2007		3025.6	3675.0	0.21
2008		3764.3	2758.4	(0.27)
2009		2789.8	2533.5	(0.09)
2010		2560.6	2373.6	(.07)
2011		2395.8	1995.1	(0.17)
2012		2002.5	1957.6	(0.02)
2013	Q1	**1958.5**	2101.4	0.07
	Q2	2119.9	1980.5	(0.07)
	Q3	1972.0	1850.6	(0.06)
	Q4	1869.9	2065.8	0.10
2014*	Q1	2086.8	2148.9	0.03
	Q2	2177.9	2115.8	(0.03)
	Q3	2113.9	2115.0	0.00
	Q4	2118.9	2165.5	0.02

Source: Arab Federation of Exchange Report 2012-2014 and ASE- collected by the researcher

In the second quarter of 2014, the ASE index lost 3% of its market value (Dabbas, 2013). Dabbas (2013) explained that this was due to: i) the decline in the first quarter profit of listed companies, compared with the same period of the previous year, especially the leading companies like Arab Potash Co., Jordan Phosphate Co., Arab Bank, and Jordan Telecom; ii) limited movement of non-Jordanian investments, despite their values being more than half of the market value of the listed companies; iii) limitation in the retention period of the purchased shares, which is considered as an indicator of continuous uncertainty and uneasiness on the decisions of non-Jordanian investors; iv) negative impact of speculators on market movement resulting from their concentration on speculative stocks; v) weak institutional investment, and vi) declining daily trade values, which lead to higher investment risks as a result of the high level of price fluctuations that reflects a reduction in speculation activity and widespread rumours in the market.

As shown in Table 1.2, the Jordan economy has been affected and is still suffering from the 2007 global economic crash. This is demonstrated by the ASE indicators that recorded the decrease in liquidity flows, market value, and Gross Domestic Product (GDP) by 75.5%, 37.6%, and 71.3%, respectively in 2013 as compared to 2007. It is also noticed that there was an increase of 72.6% in the total deposits in banks for the same period. Responding to this situation, some investors withdrew from the market, while others refrained from trading and/or become more careful in selecting their investments.

The Amman Stock Exchange situation creates a motive to extend the SGMIP model in solving the portfolio selection problem and retailor it to absorb new constraints that are practically used by investors, and to make it mathematically solvable. To investigate the portfolio elements, the number of assets to hold, transaction cost, liquidity, and information on selecting a dynamic portfolio under uncertainties should be considered. In this study, uncertainty is represented in terms of securities price. An attempt has been made to apply and extend the mathematical model that simultaneously generates the optimal portfolio and suits the markets that have the same characteristics of ASE.

Table 1.2: ASE Financial Indicators.

Year	Trading Volume (Million JD)	Trading Market Value (Million JD)	No. Of Shares	No. of Transaction	Index	Total Market Value Billion JD	Market Capitalization / GDP (%)	Total Deposit Volume (Million)
2005	16,871.0	26,667.0	2,581,744,423	2,392,509	4259.7	26.7	326.6	13119.3
2006	14,209.8	21,078.2	4,104,285,135	3,442,558	3013.7	47.7	233.9	14591.9
2007	12,348.1	29,214.2	4,479,369,609	3,457,915	3675	77.0	289	15988.1
2008	20,318.0	25,406.2	5,442,267,689	3,780,934	2758.4	102.4	216.7	18102.6
2009	9,665.3	22,526.9	6,022,471,335	2,964,610	2533.5	124.9	149.6	20298.4
2010	6,689.9	21,858.1	6,988,585,431	1,880,219	2373.6	146.8	122.7	22504.8
2011	2,850.2	19,272.7	4,072,337,760	1,318,278	1995.1	166.0	102.7	24377.9
2012	1,978.8	19,141.5	2,384,058,415	975,016	1957.6	185.2	93.5	24969.7
2013	3,027.2	18,233.4	2,705,796,950	1,074,438	2065.8	203.4	83.01	27593.2
2014	2,263.4	18,082.6	2,321,802,789	955,987	2165.5	200.1	75.75	30261.0

Source: Jordan Security Commission, Annual Reports 2005-2014.

This chapter is organized as follows: Section 1.1 is an overview, Section 1.2 defines and explains research problem, Section 1.3 formulates the research questions, objectives of the study are described in Section 1.4, a scope of the study is presented in Section 1.5, this is followed by Section 1.6, that provides significance of the study, Section 1.7 displays the definition of terms, and finally Section 1.8, is the summary of the chapter.

1.2 Research Problem

In 2013, the two leading companies of Amman Stock Exchange, i.e., Arab Potash Company (APOT) and Jordan Phosphate Company (JOPH), have lost 39.7% and 45%

of their market price, respectively. The closing price of APOT on January 2nd 2013 was 46.510 JD and was 28.050 JD on December 31th 2013, which reflected a decrease of 39.7%. JOPH's closing price decreased from 13 JD to 7.150 JD, representing a decrease of 45% during the same period (ASE Annual Report, 2013).

Arab Potash Company (APOT) and Jordan Phosphate Company (JOPH) are just two recent examples of the uncertainty facing ASE since years ago. If one thinks about the large number of similar examples in the market, the problem of picking a profitable security only turns to be more complex. Regardless of the reasons behind the drop in prices of these companies, even an expert portfolio manager could make a poor investment decision in such conditions; hence, proving that the portfolio selection is complex and dynamic (Stoyan, 2009). Therefore, portfolio managers and investors have become more careful in assets selections and allocations in developing a secure and structured portfolio as their attempts to outperform the frequent market failures.

In view of that, investors trading in such conditions within ASE are interested in selecting their optimal portfolio in order to minimize the portfolio risk. Like other investors, ASE's investors are facing the portfolio selection problem in finding a combination of assets that best satisfies their needs. This problem has three dimensions. First is the multi-assets problem in selecting a set of securities to invest in. In line with this, adding new financial instruments and derivatives to an existing portfolio increase the complexity of the problem and make the role of the most skillful and experienced portfolio manager harder. Under uncertain conditions, bond investments seem to be a reasonable financial option for many portfolio managers because of their steady cash flow. Bonds have fixed maturities and their prices are based on the fluctuation of interest rates, which make them different from stocks. Bonds are subjected to a degree of uncertainty that pose an inverse relationship with that of stocks (Konno & Kobayashi, 1997), which makes it a viable investment option for an investor in uncertain circumstances. Therefore, investigating a model involving stock and bond portfolio selection appears of great relevance. Second is the multi-objectives problem, where the risk-return portfolio is traded off between two practical constraints. There are many factors or practical constraints that spark interest of investors to include in their portfolio to improve the decision in selecting the optimal portfolio such as limiting the portfolio size and transaction cost, at the same time, encouraging liquidity, diversity, and information. Investors aim to satisfy their interests and needs, the more these needs is increased, the more complexity of the problem is, and the role of the investor becomes more difficult (Ji, Zhu, Wang & Zhang, 2005; Ibrahim, 2008; Stoyan, 2009; Stoyan & Kwon, 2010; Stoyan & Kwon, 2011; Brown & Smith, 2011; Moallemi & Sa˘glam, 2013; He & Qu, 2014**)**. At last is the multi-period problem that exists as an investor typically makes investment decisions for future period; the current information is already reflected in the market prompting the use of historical data in the estimation. Making predictions under uncertain environments will increase the variation or “drifts”. Moreover, uncertain conditions require periodic rebalancing of the portfolio or when adapting with new circumstances (Valian, 2009).

As a consequence, since 1952, efforts have been made to design schemes or approaches, which could reserve the potential of investors to simplify and discover new dimensions of the utilised schemes. These efforts have contributed to new dimensions and provided new directions in this field. Many attempts have been taken to develop models to deal with portfolio problems such as MV that only considers up-side returns, generates portfolios that are limited by a single period approach, and has implementation issues, and apparently, as it turns out, risk and covariance are not equivalent measures (Markowitz, 1952; Konno & Yamazaki, 1991; Feinstein & Thapa, 1993; Grootveld & Hallerback, 1999; Chang et al., 2000; Jobst et al., 2001; Karacabey, 2007; Li, Qin & Kar, 2010; Liu & Zhang, 2015; Farias, Vieira, & Santos, 2015). Numerous attempts have been made to consider down-side risk via the Value at Risk (VaR) method (Siwak, 2010; Liu & Qin, 2012; Ekhtiari, 2014), as well as the Conditional Value at Risk (CVaR) (Uryasev, 2000; Pflug, 2001; Tasche, 2002; Sarykalin, Serraino & Uryasev, 2008; Fabozzi, Huang & Zhou, 2010).

In addition, other researchers have used mathematical and programming techniques such as SP (Muhlemann, Lockett & Gear, 1978; Dantzig & Infanger; 1993; Yu, Ji & Wang, 2004; Abdelaziz, Aouni & El Fayedh, 2007; Ibrahim, 2008; Calafiore, 2009; Skaf & Boyd, 2009; Pang & Hussain, 2017) to capture uncertainty. Moreover, Mixed Integer Programming (Mansini & Mansini, 1999; Sağlam & Benson, 2017) and Stochastic Mixed Integer Programming (Tapaloglou, Vladimirou & Zenios, 2008; Valian, 2009; He & Qu, 2014; Moallemi & Sa˘glam, 2017) have been applied to combine uncertainty with maintaining no fraction in the solution, whereas, Goal Programming (GP) (Charnes & Cooper, 1959; Mansour, Rebai & Aouni, 2007), and Stochastic Goal Programming (SGP) (Heras & Aguado, 1999; Ballestero, 2000; Al-Zahrani & Ahmad, 2004; Ji et al., 2005; Ballestero, 2005; Aouni, Ben Abdelaziz, & Martel, 2005; Van Hop, 2007; Brown & Smith, 2011) and SGMIP (Stoyan, 2009; Stoyan & Kwon, 2010, 2011) have been employed to incorporate the investors' multi-objectives considering uncertainty with integer outcomes. Unfortunately, none of these studies are able to justify that their model is the most effective in capturing the factors and optimally solving the dynamic portfolio problem. At the same time, there are limited publications involving financial models. Hence, the extended SGMIP model presented in this study is an innovative contribution to the field.