



**Optimization of Middle Control Points of Cubic Bézier  
Curve for Handwriting Representation using Whale  
Optimization Algorithm**

by

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## LIST OF ABBREVIATIONS

ABC	Artificial Bee Colony algorithm
AI	Artificial Intelligence
CAD	Computer Aided Design
CAGD	Computer Aided Geometric Design
CS99	Chetverikov and Szabo algorithm
DE	Differential Evolution algorithm
DIA	Document Image Analysis
FD77	Freeman algorithm
GA	Genetic Algorithm
LSA	Least Square Approximation
MATLAB	Matrix Laboratory
NFL	No Free Lunch theorem
OCR	Optical Character Recognition
SAM06	Sarfraz Asim and Masood algorithm
SSE	Sum of Squared Error
WOA	Whale Optimization Algorithm
WOA_cg_curve	Whale Optimization Algorithm convergence curve
WOA_CP	Whale Optimization Algorithm for Control Point
2D	Two Dimensional
3D	Three Dimensional

## LIST OF SYMBOLS

$\vec{A}$	Coefficient vector A
$\vec{a}$	Vector a
$b$	Constant value defined logarithmic shape
$\vec{C}$	Coefficient vector C
$\vec{D}$	Distance between ith whale to the best candidate (exploitation phase)
$\vec{D}'$	Distance between ith whale to random whale (exploration phase)
$F$	Sharpness of opening angle
$L$	Length of $P_iP_k$
$l$	Random value in range [-1,1]
$N$	Population size or number of search agent
$P$	Original boundary
$p$	Probability to choose between two mechanisms
$Q(s)$	Reconstructed parametric curve
$R$	Range of candidate corner
$\vec{r}_1$	Random value in range [0,1]
$\vec{r}_2$	Random value in range [0,1]
$s$	Parameter of chord length parameterization
$t$	Iteration
$t_{\max}$	Maximum iteration number
$\vec{X}$	Current position of whale
$\vec{X}_{best}^*$	Current best position of whale
$\vec{X}_{rand}$	Random position of whale

## **Pengoptimuman Titik Kawalan Tengah Lengkung Bézier Kubik Untuk Perwakilan Tulisan Tangan Menggunakan Algoritma Pengoptimuman Ikan Paus**

### **ABSTRAK**

Kajian ini melibatkan isu perwakilan lengkung imej tulisan tangan menggunakan kaedah pengiraan iaitu Algoritma Pengoptimuman Ikan Paus (APIP). APIP ialah algoritma meta-heuristik baru sifatnya terinspirasi dengan meniru kelakuan sosial ikan paus kelasa. Keputusan pengoptimuman berdasarkan masalah pengoptimuman struktur dan matematik sebelum ini membuktikan bahawa APIP sangat kompetitif berbanding dengan kaedah pengoptimuman lain. Teknik perwakilan tulisan tangan memainkan peranan yang penting bagi mengiktiraf watak-watak tulisan tangan. Ketepatan perwakilan lengkung akan memberi kesan kepada keputusan pengiktirafan. Oleh itu, kajian ini berkaitan dengan proses lengkung yang sesuai menangani isu ketepatan perwakilan lengkung. Lengkung tulisan tangan perwakilan melibatkan proses lengkung penyuaian iaitu pengestrakan sempadan, pengesanan titik sudut, pemparameteran dan penyuaian lengkung parametrik. Objektif penyelidikan adalah penghampiran dengan lengkung tulisan tangan asal dengan mengoptimumkan lokasi titik kawalan pertengahan lengkung Bézier kubik. Titik kawalan pertengahan akan berubah bentuk keseluruhan setiap lengkung Bézier. Oleh itu, pada fasa terakhir lengkung penyuaian, penggunaan algoritma APIP adalah untuk mengenal pasti pengoptimuman titik kawalan tengah lengkung Bézier kubik supaya mempunyai anggaran tepat lengkung tulisan tangan asal. Jumlah Ralat Kuasa Dua (JRKD) telah digunakan untuk mengira ralat antara kedua-dua lengkung tulisan tangan iaitu asal dan lengkung berparameter baru. Dengan menggunakan titik kawalan tengah yang dipilih, optimum licin lengkung yang mengurangkan JRKD antara data asal dan data perwakilan baru kemudiannya diperolehi. Keputusan lengkung penyuaian ke atas 191 sempadan tulisan tangan dengan menggunakan algoritma APIP yang telah dibandingkan dengan kaedah langsung penyuaian lengkung yang dikenali sebagai Penghampiran Paling Kurang Kuasa Dua (PPKKD). Ia mengesahkan bahawa algoritma APIP mempunyai prestasi yang lebih baik dalam penyuaian lengkung berparameter berbanding kaedah PPKKD.

## **Optimization of Middle Control Points of Cubic Bézier Curve for Handwriting Representation using Whale Optimization Algorithm**

### **ABSTRACT**

This study concerns the issue of curve representation of the handwriting image using a computational method namely the Whale Optimization Algorithm (WOA). The WOA is a novel nature-inspired meta-heuristic algorithm that mimics the social behavior of humpback whales. Optimization results based on structural and mathematical optimization problems previously proved that the WOA very competitive compared to the state-of-the-art optimization methods. The handwriting representation technique plays an important role in the recognition of handwriting characters. The accuracy of curve representation will affect the recognition results. Thus, this study deals with a curve fitting process to handle the accuracy issue of the curve representation. The handwriting curve representation involved a process of curve fitting which are boundary extraction, corner point detection, parameterization and parametric curve fitting. The research objective is to approximate the original handwriting curve by optimizing the location of middle control points of the cubic Bézier curve. The middle control points will change the whole shape of each Bézier curve. Thus, at the last phase of curve fitting, the WOA identifies the optimum middle control points of the cubic Bézier curve. These points give an accurate approximation of the original handwriting boundary curve. Sum of Squared Error (SSE) has been used to calculate the error between the two curves namely original handwriting and the new parametric curves. By using the chosen middle control points, the optimum smooth curve that minimizes the SSE between the original data and new representation data is then obtained. The curve fitting results of 191 handwriting boundaries by using the WOA have been compared with the direct method of curve fitting known as Least Square Approximation (LSA). It is confirmed that the WOA has a better performance in fitting the parametric curve compared to the LSA method.

# CHAPTER 1 : INTRODUCTION

## 1.1 Chapter Overview

This chapter presents the research background in Section 1.2. Then, this chapter presents the problem statement and research objectives in Section 1.3 and Section 1.4, respectively. Research scope and significance of the study are also discussed in Section 1.5 and Section 1.6, respectively. Finally, this chapter is ended with a thesis outline.

## 1.2 Research Background

Writing is the style of describing something by a set of signs, symbols or alphabets known as a “writing system”. There are many writing systems all around the world, for instance Latin, Arabic and Greek. Writing becomes one of the most important ways for communication, expression and information in daily life.

In the beginning, the first document was produced manually with handwriting. These historical documents written by well-known scientists, artists, writers and others represent a considerable value but remain very fragile. Later, documents were produced using modern technology by printing text on paper. These documents can be digitized and world-wide distributed in digital version using web-based portals due to modern technologies. The original documents can be conserved and protected. Generally, digital documents are in the form of an image resulting from the scanning process. It is not directly understandable by a computer-assisted application for text documents, mostly when it deals with handwriting. As technology arises, there is a computer assisted application known as recognition system. It helps to recognize the characters in

the handwriting script document. However, there is still an unsolved problem in understanding or recognizing the handwriting script's character since a lot of information loss at the early stage of recognition such as detection of significant corner and optimal control points (Singh, Sharma, & Chhabra, 2017). It is due to the unconstrained, cursive and noisy nature of the handwriting.

A significant problem in computer vision is that of object recognition and scene understanding. The objects shown in the images are often represented in terms of boundaries, surfaces and other image characteristics. Specifically, the conciseness of data representation and boundary description accuracy make boundary-drawing preferred in many circumstances. The boundary lines are considered one of the essential features of an object shown in an image.

To obtain the best result of recognition, this study needs to achieve a high accuracy approximation of the original images or objects at the early recognition process stage. Thus, this study focuses on approximating the original image of the handwriting by using an automated technique, the Whale Optimization Algorithm (WOA). The approximating activity of the boundary image can be known as curve fitting. The main idea is to represent or approximate the original boundary of the handwriting at an optimal solution. In this case, the optimal solution shows that this study obtained a good dataset of control points. Thus, the error of differences between the original and new representation boundaries can be minimized.

Curve fitting is the vital technique used to reconstruct or represent boundary curves from partial information (such as boundary samples). The curve fitting task is to construct a smooth curve that fits a set of given points in the space (such as corner points). The curve fitting objective is to use a minimum number of curve pieces to approximate the object's boundary lines with minimum misrepresentation. It is widely used in digital image analysis, which is boundary extraction and shape representation.

Conventionally, the data fitting problem solves by using direct or classical optimization algorithm methods such as data linearization and Linear Square Approximation (LSA). However, the LSA method is the most widespread approximation scheme. It was used early on in most industries and academicians include (Hayes & Halliday, 1994; Rusdi & Yahya, 2015b; Sarkar & Menq, 1991). The LSA method lends itself to the inclusion of conditions that aim at the shape of the result, not just at the closeness of fit. These conditions are typically the result of minimizing certain functions.

However, non-systematic and massive data are challenging to be solved directly. Accordingly, soft computing method's performance draws researcher's attention to solve the curve fitting problem. Implementing a soft computing method not only aids in finding an optimal solution but computationally efficient in solving any problem. This study proposed a scheme of WOA using cubic Bézier curve and to study its performance in handling with curve fitting problem. The application of the curve fitting process on the handwriting image has been made throughout this study.

The WOA is a swarm-based algorithm introduced for continuous optimization problems. It is considered that whales are the most intelligent animals with motion. Eventually, they can create their own dialect or behaviour, especially hunting their prey known as bubble-net attacking. The unique hunting behaviour of humpback whales inspired the WOA (Mirjalili & Lewis, 2016). It is recognized that this algorithm has a better or comparable performance as compared to some existing algorithmic techniques (Mirjalili & Lewis, 2016).

The curve fitting problem continually involved accuracy and efficiency (Gálvez & Iglesias, 2013). Meanwhile, the application of the WOA using cubic Bezier curve has not been discussed so far in optimizing accuracy of curve fitting. Thus, this study has been decided to explore the performance of the WOA using cubic Bézier curve in solving the handwriting curve fitting problem concerning real and complex data. It has been used to identify the middle control points of each segment of the handwriting. Sum of Squared Error (SSE) has been used to evaluate how well the solutions (the optimal middle control points obtained) represent a new parametric curve.

### **1.3 Problem Statement**

A best fitted curve can be used to aid data visualization to approximate values of a function where no data are accessible and to analyze relationships among two or more variables. So, it involved an approximation method to reconstruct a curve that represents the original data points. Boundary representation of an object is a practice to preserve the complete shape of an object (Sarfranz, Irshad, & Hussain, 2013).

Generally, the result obtained by using the LSA method is good and acceptable. Although it has been used in industry or engineering fields, the problem still occurs regarding the accuracy and efficiency, especially when dealing with real, big, and complex data. In general, the data points in reverse engineering are often noisy. It makes the LSA of the data has its potential drawback such that it is sensitive to the outliers or noises because of the overfitting problem (Ling & Li, 2005). Correspondingly, Iglesias, Galvez, and Collantes (2016) stated that the LSA could not solve a highly nonlinear and multivariate continuous optimization problem. Thus, it will affect the accuracy of curve fitting problems. Besides, approximating measured data by a smooth curve is a frequent problem in computer aided design, image processing, motion analysis and other related fields (Ismail, 2014).

Since then, over the last two decades, soft computing methods are gaining attention in the field of Computer Aided Geometric Design (CAGD). This study employed soft computing methods to gain an optimum solution to their data representation problem within a short computational time. However, only some studies have provided promising results. Either the complexity of adjusting parameter or the algorithm itself affects the result. The curve fitting problem continually involved accuracy and efficiency (Gálvez & Iglesias, 2013). Sometimes, an optimization algorithm may work best for one optimization problem, but not for other problems. This statement is supported by No Free Lunch (NFL) theorem. There is no meta-heuristics algorithm best adapted to solving all optimization problems (Wolpert & Macready, 1997).

Thus, this study aims to analyze the proposed scheme's performance, namely, the WOA using cubic Bézier curve in solving the curve fitting of the real and complex handwriting boundary. It is introduced to obtain a very accurate fitting curve to a given set of data points. The WOA has good performance in solving difficult continuous optimization problems (Mirjalili & Lewis, 2016). Despite of its valuable features, the WOA has never been applied in the context of curve fitting for geometric modeling to the best of authors' knowledge. The research aims to fill this gap. The curve fitting approach of the complex curves has been proposed. It is to establish a system that highly efficient and accurate.

#### **1.4 Research Objectives**

This study aims to obtain the parametric representation of a handwriting character that matches its digitized images using new soft computing technique. Thus, three objectives have been extracted to achieve as follows,

- a) To fit the data using the cubic Bézier curve for every segment of the handwriting boundary curve.
- b) To propose a new scheme of WOA using cubic Bézier curve for curve reconstruction.
- c) To compare the numerical and visual results of fitting handwriting boundary curve between the proposed method (WOA using cubic Bézier curve) and the direct method (Least Square Approximation).

## 1.5 Research Scope

To achieve the objectives of the study, extensive experimental studies have been carried out the reconstruction of handwriting characters is based on the theory of two dimensional (2D) curves. The handwriting representation has been implemented in Matrix Laboratory (MATLAB) R2018b.

Reconstruction of handwriting characters is based on the theory of two dimensional (2D) curves. The handwriting was written in the Latin script writing system, cursive and unconstrained. It was off-line handwriting. The characters or alphabets are written on a regular sheet of paper using a pen. The Latin-script alphabet consists of 26 letters, which can be in uppercase or lowercase alphabet as Table 1.1. The data includes the punctuation marks such as the full stop and comma. The data obtained from the online database (IAM Handwriting Database, 2019). It is an English sentence database for offline handwriting recognition.

Table 1.1 Basic Latin alphabets

	A	B	C	D	E	F	G	H	I	J	K	L	M
Uppercase	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
	a	b	c	d	e	f	g	h	i	j	k	l	m
Lowercase	n	o	p	q	r	s	t	u	v	w	x	y	z

This study carried out a handwriting boundary fitting by using a curve known as Bézier curve. The Bézier function with degree of three has been used throughout this study as it is the most flexible and stable. The curve fitting is one of the important areas in image processing. It is most often, although not exclusively, used for images of handwriting so this study describes it here in this context. Besides that, corner points of handwriting boundary were detected using Sarfraz, Asim and Masood 2006 (SAM06) algorithm.

In addition, the optimization algorithm used in this study was presented by (Mirjalili & Lewis, 2016). The parameters setting of the WOA used in this study are 30 and 100 for population size and maximum iteration number, respectively. It was an acceptable parameter setting for handwriting representation. The proposed scheme was used to optimize the middle control points of cubic Bézier curve. Furthermore, the proposed scheme was compared with direct method, LSA to assess its performance on handwriting representation. The comparison results obtained were demonstrated visually and numerically.

## **1.6 Significance of Study**

The study's findings will benefit to the curve fitting analysis, considering that the new soft computing technique to solve a curve fitting accuracy issue. Thus, the application of the WOA-cubic Bézier curve approach derived the approximation of handwriting boundary better. It is important to reconstruct the curve in an optimal solution for any purpose, such as preserving the images, recovery of missing pieces and recognition. It also can be used in Optical Character Recognition (OCR) system.

## 1.7 Thesis Outline

The master thesis contains 5 chapters: Introduction, Literature Review, Research Methodology, Result and Discussion, and Conclusion. Chapter 1: The introduction starts by broadly describing the research background, then explains the problem statement, the research objectives, scope, significance of the research and thesis outline.

Chapter 2 focuses on the literature review. The previous work from other researchers is summarized. It is divided into seven categories: handwriting, curve representation, the process of curve fitting, general shape representation by using curve, handwriting representation and the WOA. This chapter will briefly state the basic knowledge of existing techniques.

Chapter 3 describes the method that has been used in this study. Firstly, it presents a general algorithm of the parametric curve fitting process. This chapter will discuss pre-processing: boundary extraction, corner point detection and chord length parameterization. The Bézier curve also will be explained in this chapter. Then, the parametric curve fitting by using the WOA and LSA will be discussed. This chapter will present an application handwriting image. Plus, this chapter explained how the WOA works for a simple function. The objective or fitness function (SSE) used in the optimization process is also explained in this chapter.

Chapter 4 presents the results and analysis. The pre-processing results of simple, moderate and complex handwriting curve presented in this chapter. The comparison curve fitting results of the WOA and LSA method of simple, moderate and complex