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Tunable Transparency and Group Delay in Cavity Optomechanical Systems with Degenerate Fermi Gas

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Abstract: We theoretically investigate the optical response and the propagation of an external probe field in a Fabry–Perot cavity, which consists of a mechanical mode of trapped, ultracold, fermionic atoms inside and simultaneously driven by an optical laser field. We investigate the electromagnetically-induced transparency due to coupling of the optical cavity field with the collective density excitations of the ultracold fermionic atoms via radiation pressure force. Moreover, we discuss the variations in the phase and group delay of the transmitted probe field with respect to effective cavity detuning as well as pumping power. It is observed that the transmitted field is lagging in this fermionic cavity optomechanical system. Our study shall provide a method to control the propagation as well as the speed of the transmitted probe field in this kind of fermionic, ultracold, atom-based, optomechanical cavity system, which might have potential applications in optical communications, signal processing and quantum information processing.

Keywords: optomechanics; fictitious fermionic mirror; electromagnetically-induced transparency; Fabry–Perot cavity; transmission coefficient

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1. Introduction

The implementation of different aspects of quantum phenomena at the mesoscopic level has been extensively studied using cavity optomechanical systems, which couple the optical degree of freedom to the mechanical motion of a cantilever [1–4], and has vast applications in the emerging area of quantum technology such as quantum information processing [5–9], ultrahigh-precision measurement [10], gravitation-wave detection [11], quantum entanglement [12–17], nonclassical photon statistics and squeezing [18–20], optomechanically induced transparency (OMIT) [21–23] and optomechanically induced absorption (OMIA) [24–26]. In these optomechanical systems, coupling to the moving-end mirror [27–32] or mechanical membrane inside it [33–36] is obtained via radiation pressure inside a cavity as well as indirectly via quantum dots [37] or ions [38]. Furthermore, recent experimental advances have made it entirely possible [39] to couple mechanical resonators with the atomic ensembles, where the interaction is mediated by the field inside the cavity that couples the mechanical resonators with the atoms' internal [39,40] or motional [41] degrees of freedom, which may result in, for example, cooling the mechanical resonator via an atomic bath [42]. Additionally, the anti-Stokes sidebands of the stronger pump field induced by mechanical oscillation can interfere with the near-resonant probe laser inside the cavity. As a result, the optomechanical system can significantly alter the propagation of the external probe beam. The output spectrum will display a transparency window as a result, similar to an electromagnetically induced transparent (EIT) process, which had been demonstrated theoretically in [29,43] and experimentally in [4,44,45]. EIT is a quantum interference phenomena found in atoms and molecules. The electromagnetic field controls the corresponding optical response of the atomic media.

Bringing together the instruments of cavity quantum electrodynamics (QED) [46,47] and ultracold gases [48–51] opens up new opportunities for cavity optomechanics [52]. The interaction of an atom ensemble with the light mode results in an atom–light coupling being obtained in a high-finesse cavity. In the dispersive regime, this provides a large optomechanical coupling strength, tying atomic motion to the evolution of the cavity field. Moreover, the works given by [53–55] studied the optomechanical interaction inside a Fabry–Perot cavity between a light field and the mechanical mode of ultracold bosonic and fermionic atomic gases. The authors claim that the momentum side mode of ultracold atoms operates like the moving mirror of optomechanical systems in the limit of low photon numbers, regardless of the quantum statistics of the atoms. Thus, a significant coupling strength is produced between the cavity field and an ultracold atom collective density excitation that matches the cavity mode. Moreover, we would like to mention here that the bosonic condensate dynamics are often dominated by atomic collisions and collision frequency shifts that can lead to significant suppression in the effects of atom–field interactions [56,57]. This is one of the major reasons that make ultracold fermionic atom optics remain a subject of active research.

In this paper, we theoretically study electromagnetically induced transparency due to coupling of the optical field with collective density fluctuations associated with particle-hole excitations of ultracold fermionic atoms trapped inside a Fabry–Perot cavity. Here, fermionic mode serves as a mechanical oscillator, and we call this fictitious mirror a fermionic mirror. We show that the destructive quantum interference between a pump laser beam and probe beam induces a transparency window in the transmission spectra of the probe field, which can be tuned by the pump laser field. It is observed that the cavity becomes completely transparent when the amplitude of the pump laser is $E_{pu} = 0.030\,\text{MHz}$. As we increase the pump power further, the transmission spectra is significantly amplified near the resonance region. In addition, we also discuss the variations in the phase as well as group delay of the transmitted probe beam as a function of the effective cavity detuning and pumping power.

This paper is organized as follows. In Section 2, the model Hamiltonian of our proposed system, as well as its analytical solutions through input–output formalism, are described. In Section 3, we have presented the results and discussion regarding the optical response and the group delay phenomena of the output probe field. We have concluded our results in Section 4.

2. Model and Hamiltonian of the System

We consider a system that consists of N-spinless fermion atoms which are trapped in an ultra-highly finessed Fabry–Perot cavity as shown in Figure 1. The length of the cavity is L along the x-direction and is driven by a pump laser field at frequency ω_{pu} and wave number K, accompanying a weak probe laser field at frequency ω_{pr} along the cavity axis. The probing field is monitored using a detector. The Hamiltonian of the fermions-cavity system in the frame rotating at laser frequency ω_{pu} [53–55] is,

$$\begin{aligned}\hat{H} &= \hbar\Delta_o\hat{c}^\dagger\hat{c} - i\hbar E_{pu}(\hat{c} - \hat{c}^\dagger) - i\hbar E_{pr}(\hat{c}e^{i\Delta_{pr}} - \hat{c}^\dagger e^{-i\Delta_{pr}}) \\ &\quad + \textstyle\sum_k \epsilon(k)\hat{f}_k^\dagger\hat{f}_k + \frac{1}{4}\hbar U_0\hat{c}^\dagger\hat{c}\sum_k(\hat{f}_{k+2K}^\dagger\hat{f}_k + \hat{f}_k^\dagger\hat{f}_{k+2K}),\end{aligned} \tag{1}$$

where $\epsilon(k) = \hbar^2k^2/(2M)$ is the kinetic energy of a fermion and $\Delta_o = \omega_c' - (\omega_{pu} + \omega_{pr})$ is the effective cavity detuning with $\omega_c' = \omega_c + U_0N/2$, $U_o = g_o^2/(\omega_{pu} - \omega_a)$ as the coupling between atoms and the light field, where g_o is single photon Rabi frequency, ω_a is the atomic resonance frequency and ω_c the resonant frequency of the empty cavity nearest to the laser frequency. Moreover, $\hat{f}_k(\hat{f}_k^\dagger)$ is the annihilation (creation) operator of the fermion with an anti-computation relation of $\{\hat{f}_k, \hat{f}_{k'}^\dagger\} = \delta_{k,k'}$. The first term gives the energy of the intracavity field, $\hat{c}(\hat{c}^\dagger)$ denotes the annihilation (creation) operator of the cavity mode and $U_0 N/2$ corresponds to the shift in the empty cavity resonance induced by the atoms. The second and third terms correspond to the input lights with frequencies ω_{pu} and ω_{pr} with amplitudes E_{pu} and E_{pr} related to the laser power P by $|E_{pu}| = \sqrt{2P_{pu}\kappa/\hbar\omega_{pu}}$ (κ is the decay rate of the cavity amplitude) and $|E_{pr}| = \sqrt{2P_{pr}\kappa/\hbar\omega_{pr}}$, respectively. Moreover, the fourth term accounts for free energy of the fermionic atoms, and the last term describes the interaction between the fermions and the intracavity field.

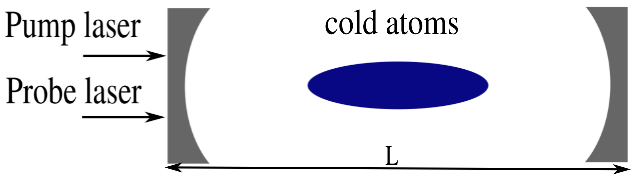

Figure 1. A sample of two-level fermionic atoms with resonant frequency ω_a trapped inside a Fabry–Perot cavity of length L. The cavity is simultaneously driven by the strong pump laser field of frequency ω_{pu} and weak probe laser field of frequency ω_{pr}. Here, the left-end mirror is transmissive while the right-end mirror is perfectly reflecting.

Hereby we consider an interaction between two-level spinless fermionic atoms with a standing-wave light field and Fermi momentum k_F. It gets a momentum kick of $2K$ at $k > 0$ due to photon recoil, and a conjugate momentum kick of $-2K$ at $k < 0$. The atomic-fluctuation operator $\hat{\rho}_p$ is then introduced and defined as

$$\hat{\rho}_p = \sum_k \hat{f}_k^\dagger \hat{f}_{k+p}, \tag{2}$$

which explains the superposition of particle-hole excitation with an excitation momentum of $2\hbar K$ for $k > 0$. In addition, we also define a $\hat{\rho}^{(+)}$ right- and $\hat{\rho}^{(-)}$ left-propagation atomic density operator that results from the summation of k for $k > 0$, and $k < 0$ [58], respectively. Therefore, by introducing the operators,

$$\begin{aligned} \hat{b}_p &= \beta_p \hat{\rho}_p^{(+)}, \qquad \hat{b}_p^\dagger = \beta_p \hat{\rho}_{-p}^{(+)}, \\ \hat{b}_{-p} &= \beta_p \hat{\rho}_{-p}^{(-)}, \qquad \hat{b}_{-p}^\dagger = \beta_p \hat{\rho}_p^{(-)}, \end{aligned} \tag{3}$$

where $p = 2K$ and the normalization constant is $\beta_p = \sqrt{2\pi/pl}$, it can be easily shown that $\hat{\rho}_p^\dagger = \hat{\rho}_{-p}$. This new operator follows commutation relations $[\hat{b}_{\pm p}, \hat{b}_{\pm p}^\dagger] = 1$, and $[\hat{b}_{\pm p}, \hat{b}_{\pm p}] = [\hat{b}_{\pm p}, \hat{b}_{\mp p}^\dagger] = 0$.

Assuming perfectly degenerate fermions and a large number of atoms such that $K < k_F = \pi N/L$, we can write the expression for quadratic energy dispersion about the Fermi energy as

$$\sum_k \epsilon(k) \hat{f}_k^\dagger \hat{f}_k \simeq \sum_k v_f \hbar |k| \hat{f}_k^\dagger \hat{f}_k, \tag{4}$$

where $v_f = \hbar k_F/M$ is the Fermi velocity. We may rewrite the expression for energy using the above equation and the operator commutation relations as

$$\sum_k \epsilon(k) \hat{f}_k^\dagger \hat{f}_k \to \sum_{p>0} v_f p\hbar(\hat{b}_p^\dagger \hat{b}_p + \hat{b}_{-p}^\dagger \hat{b}_{-p}) \tag{5}$$

and by substituting Equation (2) in Equation (3), the last term in Equation (1) can be written as

$$\sum_k (\hat{f}^\dagger_{k+2K}\hat{f}_k + \hat{f}^\dagger_k \hat{f}_{k+2K}) \rightarrow (b^\dagger_p + b_p) + (b^\dagger_{-p} + b_{-p}) \tag{6}$$

Therefore, by substituting Equations (5) and (6) in Equation (1), the final effective Hamiltonian is

$$\begin{aligned}\hat{H}_{\text{eff}} &= \hbar\omega_m(\hat{b}^\dagger_p\hat{b}_p + \hat{b}^\dagger_{-p}\hat{b}_{-p}) + \hbar[\Delta_o + g(\hat{b}^\dagger_p + \hat{b}_p) + g(\hat{b}^\dagger_{-p} + \hat{b}_{-p})]\hat{c}^\dagger\hat{c} \\ &\quad -i\hbar E_{pu}(\hat{c} - \hat{c}^\dagger) - i\hbar E_{pr}(\hat{c}e^{i\Delta_{pr}} - \hat{c}^\dagger e^{-i\Delta_{pr}}).\end{aligned} \tag{7}$$

The Bose–Einstein condensate's effective Hamiltonian and the fermionic system's effective Hamiltonian are identical, with the exception that the ground state in the fermionic system is filled with a Fermi sea, and the momentum side mode corresponds to $|k| \rightarrow |k + 2K|$ rather than $0 \rightarrow |2K|$ as for bosons [48]. This is due to the fact that each particle-hole pair is interacting with a single photon, which causes bosonic excitation. As a result, there is an analogy between a mechanical fermion mode and a bosonic condensate for low photon numbers. In the former, momentum side-mode excitation is superimposed, whereas in the latter, particle-hole excitation is superimposed. Moreover, the effective Hamiltonian resembles the Hamiltonian of the cavity optomechanics [54], with a mechanical oscillator of frequency $\omega_m = 2Kv_f$ and effective optomechanical coupling $g = U_o/4\beta_p$ for a fictitious fermionic mirror.

In the following, the temporal evolution of an annihilation operator of the intra-cavity field ($\hat{c}$) and position operator of the fermionic mirror ($\hat{q}_m$) are determined using the Heisenberg equations of motion with the commutation relations $[\hat{c}, \hat{c}^\dagger] = 1$ and $[\hat{b}_{\pm p}, \hat{b}^\dagger_{\pm p}] = 1$. Furthermore, we have also introduced the quadratures of the fermionic mirror from the operators $\hat{b}$: $\hat{q}_m = \hat{q}_+ + \hat{q}_-$, $\hat{p}_m = \hat{p}_+ + \hat{p}_-$, where $\hat{q}_\pm = (\hat{b}_{\pm p} + \hat{b}^\dagger_{\pm p})/\sqrt{2}$ and $\hat{p}_\pm = (\hat{b}_{\pm p} - \hat{b}^\dagger_{\pm p})/\sqrt{2}$, respectively. We hereby deal with the mean response of the system to the probe field in the presence of the coupling field, and assume $\langle\hat{c}\rangle$, $\langle\hat{c}^\dagger\rangle$ and $\langle\hat{q}_m\rangle$ are mean values of the operators $\hat{c}$, $\hat{c}^\dagger$ and $\hat{q}_m$, respectively. The equations of motion corresponding to the mean value operators are

$$\begin{aligned}\frac{d\langle\hat{c}\rangle}{dt} &= -(\kappa + i\Delta_o)\langle\hat{c}\rangle - i\sqrt{2}g\langle\hat{q}_m\rangle + E_{pu} + E_{pr}e^{-i\Delta_{pr}t}, \\ \frac{d^2\langle\hat{q}_m\rangle}{dt^2} + \gamma_m\frac{d\langle\hat{q}_m\rangle}{dt} &+ \omega_m^2\langle\hat{q}_m\rangle = -2\sqrt{2}\omega_m g\langle\hat{c}\rangle\left\langle\hat{c}^\dagger\right\rangle.\end{aligned} \tag{8}$$

Here, fermionic and optical dissipation are taken into account by introducing the damping of the fermionic mirror γ_m and cavity decay rate κ. Under intense laser driving and weak probe-field conditions, in order to solve the equations for the expectation values of these operators, we can linearize these operators as a sum of their steady-state mean values and an additional small fluctuation around them, i.e., we make the ansatz up to the first order sideband only as follows [59–61]:

$$\begin{aligned}\langle\hat{c}\rangle &= c_o + c_1 e^{-i\Delta_{pr}t} + c_2 e^{i\Delta_{pr}t}, \\ \langle\hat{q}_m\rangle &= q_{mo} + q_{m1}e^{-i\Delta_{pr}t} + q_{m2}e^{i\Delta_{pr}t}.\end{aligned} \tag{9}$$

By substituting Equation (9) in Equation (8), comparing the same time-dependent terms on both sides and working to the lowest order in E_{pr} but to all order in E_{pu}, we get

$$c_1 = E_{pr}\left[\frac{(\kappa - i\Delta_{pr}) - i(\Delta_o - A)}{(\kappa - i\Delta_{pr})^2 + (\Delta_o - A)^2 - B}\right] \tag{10}$$

where,

$$\begin{aligned} A &= \frac{2g^2|c_o|^2}{\omega_m}\left[1+\frac{\omega_m^2}{\omega_m^2 - i\Delta_{pr}\gamma_m - \Delta_{pr}^2}\right], \\ B &= \frac{4g^4\omega_m^2|c_o|^4}{(\omega_m^2 - i\Delta_{pr}\gamma_m - \Delta_{pr}^2)^2}, \\ c_o &= \frac{E_{pu}}{\kappa + i\Delta}, \\ \Delta &= \Delta_o + g q_{mo}, \\ q_{mo} &= -\frac{g\sqrt{2}|c_o|^2}{\omega_m}. \end{aligned}$$

Moreover, to study the optical property of this system, we use the input–output relation $c_{out} = \sqrt{2\kappa}\, c - c_{in}$ [62], where c_{in} and c_{out} are input and output operators, respectively. The expectation value of the output field is

$$\begin{aligned} \langle c_{out}\rangle &= (E_{pu}/\sqrt{2\kappa} - \sqrt{2\kappa}c_o)e^{-i\omega_{pu}t} + (E_{pr}/\sqrt{2\kappa} - \sqrt{2\kappa}c_1) \\ &\quad \times e^{-(i\omega_{pu}+\Delta_{pr})t} - \sqrt{2\kappa}c_2 e^{-(i\omega_{pu}-\Delta_{pr})t}. \end{aligned} \tag{11}$$

The probe power transmission coefficient is defined as the ratio of the probe power coming from the system and the input probe power at probe frequency. Therefore, the general expression for the probe power transmission is given by

$$T_r = \frac{E_{pr}/\sqrt{2\kappa} - \sqrt{2\kappa}c_1}{E_{pr}/\sqrt{2\kappa}} = 1 - 2\kappa c_1/E_{pr}. \tag{12}$$

3. Discussion

For numerical illustration, we consider experimentally achievable parameters $U_0 = 2\pi \times 20$ kHz, $\lambda = 500$ nm ($K \simeq 10^7$ m^{-1}), $L = 100$ μm, and $N \simeq 5000$ atoms yielding a Fermi momentum of $k_F \simeq 10^8$ m^{-1}, so that $k_F = 12.5K$, $\kappa = 2\pi \times 1$ MHz, $M = 1.5 \times 10^{-25}$ kg, and the Fermi frequency is $\omega_F \equiv \epsilon_F/\hbar \simeq 10$ MHz. This value of U_0 assumes a single-photon Rabi frequency of $g_0 = 2\pi \times 10$ MHz and a pump-atom detuning of $\omega_{pu} - \omega_a = 2\pi \times 30$ GHz, similar to [48,49]. In Figure 2, we plot the transmission coefficient $|T_r|^2$ of the probe field as a function of the effective probe-cavity detuning $\Delta'_{pr} = \omega_{pr} - \omega'_c$ for different values of the pump field $E_{pu} = 0, 0.02, 0.03$, and 0.035 MHz, respectively. In the presence of the pump laser, the transmission spectrum of the probe beam shows a significant transparency window when resonance condition, i.e., ($\Delta'_{pr} = 0$) is met, as shown in Figure 2 for the series of the pump field E_{pu}. We can clearly see that depth and width of the transparency window of the probe field are fully controllable via the pump laser. Therefore, the transparency window in our system is effectively modulated with the pump laser-like conventional optomechanical system earlier demonstrated in [4,44,45]. In addition, the probe beam does not transmit through the cavity when the pump laser field is off, as shown in Figure 2. However, a significant enhancement in the transmission of the probe laser beam is obtained around the resonance region ($\Delta'_{pr} \simeq 0$) when the pump laser field is switched on. From Figure 2, it is clear that the probe beam is completely transmitted from the cavity when the pump laser field is $E_{pu} = 0.03$ MHz. This phenomena is very similar to the EIT phenomena in conventional atomic systems, and the underlying physical explanation is as follows: the collective density fluctuation of the fermions is an analogy to the moving mirror with resonance frequency ω_m. The simultaneous presence of both the applied light field and the probe field generates a radiation pressure force oscillating at frequency Δ'_{pr}, which derives the collective density of the fermions near its resonance frequency. If the beat frequency (Δ'_{pr}) is close to the resonance frequency (ω_m) of the collective density of the fermions, this gives rise to Stokes and anti-Stokes scattering from the strong intracavity field. The probe field interferes with anti-Stokes at resonance ($\Delta'_{pr} \simeq 0$); as a result, the transmission spectrum is modified [44].

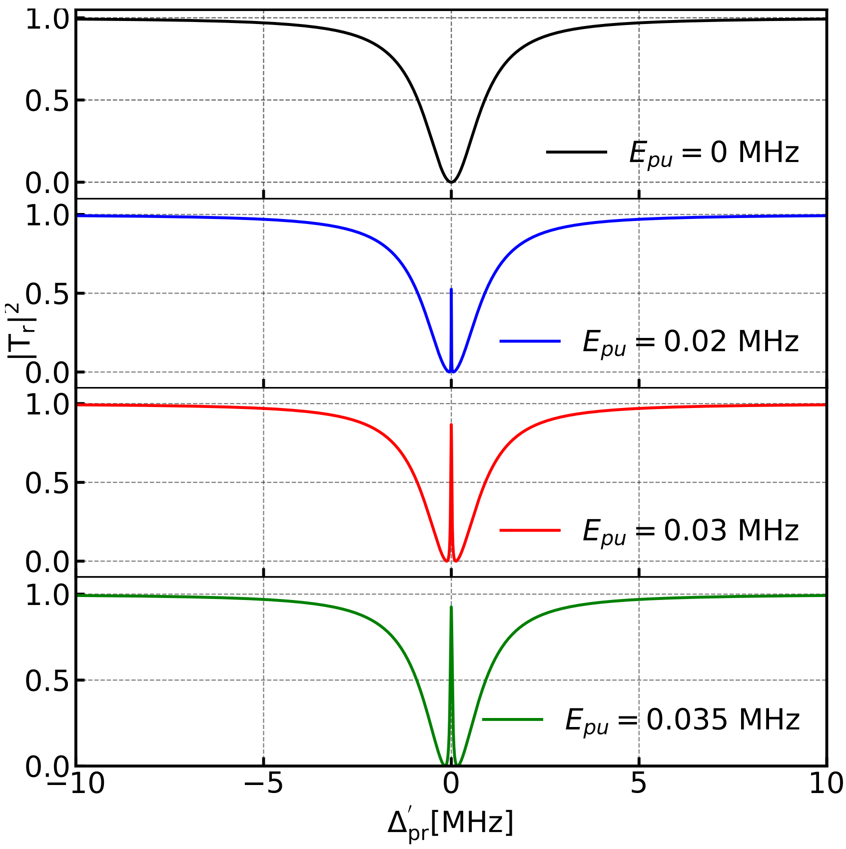

Figure 2. Plot of transmission coefficient $|T_r|^2$ of the probe laser power as function of the effective probe-cavity detuning $\Delta'_{pr} = \omega_{pr} - \omega'_c$ when pump laser field $E_{pu} = 0$ (black), 0.02 (blue), 0.03 (red) and 0.035 (green) MHz, respectively. Other parameters are $U_0 = 2\pi \times 20$ kHz, $\lambda = 500$ nm ($K \simeq 10^7$ m^{-1}), $L = 100$ μm, $N \simeq 5000, k_F \simeq 10^8$ m^{-1}, $\kappa = 2\pi \times 1$ MHz, $M = 1.5 \times 10^{-25}$ kg, and the Fermi frequency $\omega_F \equiv \epsilon_F/\hbar \simeq 10$ MHz, $g_0 = 2\pi \times 10$ MHz, and the pump-atom detuning $\omega_{pu} - \omega_a = 2\pi \times 30$ GHz.

Figure 3a is the plot of the magnified transparency window when pump power $E_{pu} = 0.0350$ MHz. The width of the transparency is several KHz. At resonance $\Delta'_{pr} \simeq 0$, the transmission coefficient has a maximum value of 1, and it rapidly goes to zero as it moves away from the resonance, as shown in Figure 3a. The underlying phenomenon of EIT is to alter the phase response of the system which ultimately modifies the propagation of light pulses [63].

The rapid phase dispersion $\phi_p(\omega_p) = \arg[t(\omega_p)]$ in the vicinity of the transparency window results in a transmission group delay in a cavity optomechanical system given as [4,22],

$$\tau_g = \frac{d\phi(\omega_{pu})}{d\omega_{pu}} = \frac{d[\arg[T_r(\omega_{pu})]]}{d\omega_{pu}}. \tag{13}$$

The variation in the group delay's magnitude is likewise altered by the rapid phase dispersion in the transmitted probe field, so $\tau_g < 0$ and $\tau_g > 0$ correspond to fast and slow light propagation, respectively [63,64].

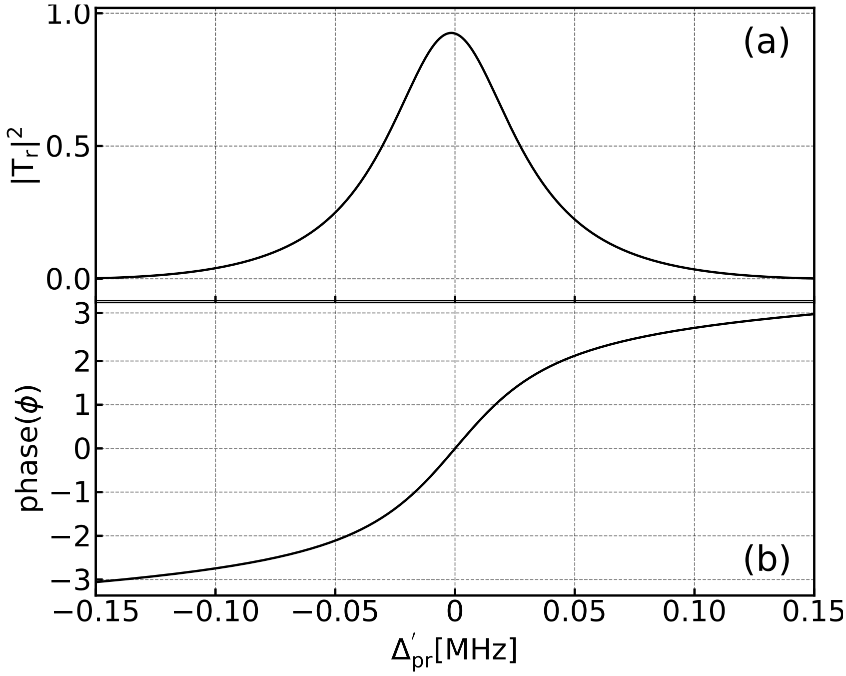

Figure 3. (**a**) Plot of transmission coefficient $|T_r|^2$ (magnified transparency window) of the probe laser power as a function of the effective probe-cavity detuning $\Delta'_{pr} = \omega_{pr} - \omega'_c$ when the pump laser field $E_{pu} = 0.035\,\text{MHz}$. (**b**) Phase plot of the transmitted probe beam as a function of the effective probe-cavity detuning Δ'_{pr} for pump laser field $E_{pu} = 0.035\,\text{MHz}$. The other parameters are the same as in Figure 2.

In Figure 3b, we plot the phase ϕ of the probe laser beam as a function of the effective probe-cavity detuning Δ'_{pr} for $E_{pu} = 0.035\,\text{MHz}$. This phase corresponds to the field transmitted from the cavity and bears a sharp increase with the increase of Δ'_{pr}, as shown in Figure 3b. In terms of group delay τ_g, as we gradually increase the pump power to $E_{pu} = 0.015\,\text{MHz}$, the group delay τ_g of the transmitted probe beam can be switched from zero to a finite negative value, and it shows slow light characteristics. However, if we increase E_{pu} beyond this, τ_g switches to a finite positive value, showing fast light characteristics approximately up to $E_{pu} = 0.020\,\text{MHz}$, and ultimately reduces to zero again, as shown in Figure 4. Hence, the propagation of the transmitted probe beam gets significantly modified in our proposed ultracold fermion-based optomechanical system.

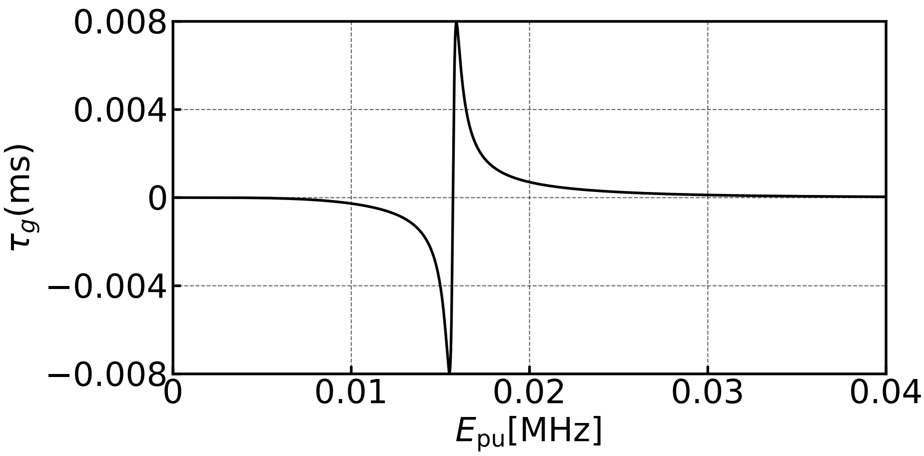

Figure 4. Group delay τ_g (in ms) as a function of E_{pu}. The other parameters are the same as in Figure 2.

4. Conclusions

In conclusion, we theoretically studied the propagation of the transmitted probe field in a system consisting of ultracold fermionic atoms trapped inside a Fabry–Perot cavity. The collective density fluctuations of the fermions associated with particle-hole excitations is analogous to the mechanical mirror. It was observed that a transparency window appeared in the transmission spectra of the probe beam at the resonance i.e., $\Delta'_{pr} = 0$, due to the interference of the stronger field with the probe beam. In the absence of the driving field, the probe beam can not transmit through the cavity, whereas when the pump laser is switched on, the probe beam can transmit through the cavity, and the cavity becomes completely transparent for $E_{pu} = 0.030\,\text{MHz}$. As we further increase the pump power, the transmission coefficient of the probe beam is significantly enhanced around the resonance condition. Furthermore, it is observed that the phase of the transmitted probe beam inside the cavity shows sharp variations with respect to effective cavity detuning Δ'_{pr}, which implies that the group delay of the transmitted probe beam τ_g can be significantly modified and altered in this fermionic mirror-cavity system for the parameter regime, very close to the work discussed in [48,49]. Finally, we hope that our present study will be observed experimentally in the near future and may provide an efficient method to control the propagation of light in trapped, ultracold, fermionic-based quantum systems.

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