Shooting-Euler Method for Two-point Fuzzy Boundary Value Problems

by

Farhah Athirah Binti Musli

(1532121576)

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Institute of Engineering Mathematics

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LIST OF ABBREVIATIONS

ADM	Adomian Decomposition Method
AGE	Alternating Group Explicit
BVPs	Boundary Value Problems
FBVPs	Fuzzy Boundary Value Problems
FDEs	Fuzzy Differential Equations
FIVPs	Fuzzy Initial Value Problems
FLT	Fuzzy Laplace Transform
GS	Gauss-Siedel
IVPs	Initial Value Problems
KPA	Kaedah Penguraian Adomian
MATLAB	Matrix Laboratory
MNSK	Masalah Nilai Sempadan Kabur
ODEs	Ordinary Differential Equations
OHAM	Optimal Homotopy Asymptotic Method
SOR	Successive Over Reaction
TAGE	Two-Parameter Alternating Group Explicit

LIST OF SYMBOLS

α	Alpha
$\in$	Element of
β	Crisp Boundary Conditions
γ	Crisp Boundary Conditions
$\widehat{\beta}$	Fuzzy Boundary Conditions
$\widehat{\gamma}$	Fuzzy Boundary Conditions
$\widehat{0}$	Fuzzy number for 0
$\widehat{1}$	Fuzzy number for 1
∞	Infinity
λ	Lambda
$\underline{y}$	Lower Branches on y
$\neq$	Not equal to
N	Number of subintervals
$\mathbb{R}$	Real Number
$\mathbb{R}_F$	Space of Fuzzy Number
$\ominus$	Symmetric difference
$\overline{y}$	Upper Branches on y

Kaedah Tembakan ke atas Masalah Nilai Sempadan Kabur Dua Titik

ABSTRAK

Persamaan pembezaan memainkan peranan yang sangat penting dalam memodelkan masalah dunia nyata. Walaubagaimana pun, kebanyakan masalah dunia nyata yang terlibat sangat sukar untuk mendapatkan gambaran yang jelas dan tepat. Oleh yang demikian, persamaan pembezaan dengan set kabur adalah diperlukan untuk mendapatkan model yang lebih baik. Dalam kajian ini, masalah nilai sempadan kabur (MNSK) dua titik dipertimbangkan dan diselesaikan dengan menggunakan kaedah Tembakan. Pertama, MNSK ditafsirkan melalui konsep kebolehbezaan teritlak dan kemudian diikuti dengan menurunkannya ke dalam bentuk parametrik, yang akhirnya menghasilkan empat kes yang berbeza. Untuk setiap kes, kaedah Tembakan digunakan untuk merangka prosedur umum untuk mendapatkan penyelesaian. Kemudian, kaedah Euler digunakan untuk menganggar penyelesaian yang sebenar. Satu contoh berangka diberikan untuk menunjukkan cara pelaksanaan prosedur yang dicadangkan. Semua simulasi berangka dilaksanakan dengan menggunakan MATLAB. Berdasarkan keputusan berangka, jelas menunjukkan bahawa anggaran penyelesaian yang dihasilkan melalui prosedur yang dicadangkan adalah sedikit berbeza dengan penyelesaian analisis yang diperolehi dengan menggunakan Kaedah Penguraian Adomian (KPA). Oleh kerana nilainya sedikit berbeza, maka dapat disimpulkan bahawa prosedur yang dicadangkan adalah berkeupayaan untuk menyelesaikan sebarang MNSK dua titik di bawah konsep kebolehbezaan teritlak.

Shooting Method for Two-point Fuzzy Boundary Value Problems

ABSTRACT

Differential equation plays an important role in modeling of real world problems. However, many real world problems involved are very difficult to obtain clear and exact model. Therefore, differential equation with fuzzy set is essential to provide better understanding. In this study, a two-point fuzzy boundary value problem (FBVP) is considered and solved numerically by using the shooting method. First, the FBVP is interpreted under the concept of generalized differentiability and then followed by reducing it into parametric forms, which finally obtained four different cases. For each case, the shooting method is used to construct the general procedure of obtaining solutions. Then, Euler method is used to approximate the final solutions. A numerical example is provided in order to show the application of the proposed procedures. All numerical simulations are executed by using MATLAB. From numerical results, it is clear that the approximate solutions obtained by using the proposed procedures are slightly different compared to analytical solutions obtained by using Adomian decomposition method (ADM). Since the values close to each other, thus it can be concluded that the proposed procedure is applicable to solve any two-point FBVP under the concept of generalized differentiability.

CHAPTER 1

INTRODUCTION

1.1 Introduction

This chapter is about introducing the matter of the research. This chapter is divided into several sections. First section is about the discussion on the overview of fuzzy set itself. Later, it followed by research background, problem statements of the research, objectives of the research, research scope and lastly is significant of research.

1.2 Overview

The concept of fuzzy set has been introduced by Zadeh (1965). According to Wang (1997), fuzzy defines as vague, blurred or imprecise of an environment as indicated by Oxford Dictionary. Since then, fuzzy is continuously discovered and developed. Nowadays, fuzzy set theory and fuzzy logics are broadly utilized for handling the issues happened in real life application. Though in the real world, it is impossible to acquire exact depiction. Thus, approximate solution is needed. Then, finding the approximate solution by using fuzzy set became important in order to get a reasonable yet practical model and solution (Wang, 1997). Furthermore, the theory of fuzzy sets itself has advanced in a variety of disciplines as it is knows as powerful tool for illustrating the uncertainty and treating vagueness in mathematical models (Chalco-Cano & Román-Flores, 2008). The applications of fuzzy system can be found in variety of field such as artificial intelligence, computer science, medicine, decision theory, management science, operations research and robotics (Zimmermann, 2010).

Fuzzy systems revolve with linear and nonlinear problem that exist in science and engineering. One of them is second order differential equation with various types of boundary value conditions which can be solved either analytically or numerically (Islam & Shirin, 2011). See, Majid, & Suleiman (2011) stated boundary value problems (BVPs) in science and engineering are applicable to appeal faster and accurate numerical method. However, not all fuzzy initial or BVPs could be solved exactly due to difficulty in finding their analytical solution (Guo, Shang, & Lu, 2013). This situation leads by considering their approximation solution as a final solution becoming more important and demanding. Examples of BVPs occur in wide variety of problems such as modeling of chemical reactions, the boundary layer theory in fluid mechanics and heat power transmission (Rahman, Ara, Islam, & Ali, 2015).

In order to solve the BVPs, numerous researchers attempt to obtain higher accuracy rapidly by using several numerical methods. A few techniques are accessible and among them are finite difference method, shooting method and finite element through projection method (Kumar, Paul, & Hoque, 2014). As for the purpose of this study, shooting method is used to solve two-point fuzzy boundary value problems (FBVPs).

1.3 Research background

The study involves the analysis of shooting method toward two-point FBVPs. Handling problems regarding on fuzzy differential equations (FDEs) and FBVPs are one of the foremost application for fuzzy number arithmetic (Buckley & Feuring, 2000).

FBVPs are approachable by two ways. The first approach stated problems in where the boundary values are fuzzy set will give the solution still in fuzzy function. Meanwhile for second approach is based on generating the fuzzy solution from the crisp solution (Gasilov, Amrahov, & Fatullayev, 2011). To solve these problems, numerical methods is used to obtain the approximate solution.

Shooting method is the most common technique used in solving problem concerning on BVPs (Morrison, Riley, & Zancanaro, 1962). Normally, Ordinary Differential Equations (ODEs) are given with initial condition or boundary condition. Basically, this method is done where the differential equations given replaced into an initial value problems (IVPs) with assuming for the unknown initial values (Adam & Hashim, 2014). The boundary value obtained is compared with the actual boundary value. Once the BVPs are transformed into such an equivalent initial value problem, then any of numerical method can be applied to find a numerical solution for the IVPs. In this study, Euler method is integrated in shooting method. Euler method is used to find the approximation solution.

1.4 Problem statements

Previously, the idea came from concept of Hukuhara differentiability which is being used to interpret two-point BVPs for second order fuzzy differential equation (Hukuhara, 1967). At first, concept of Hukuhara differentiability is applied for fuzzy number value functions (Puri & Ralescu, 1983). Furthermore, the existence and uniqueness of the solution of a FDEs were studied. Bede and Gal (2005) argued this type of differentiability shows it is not successful in obtaining the solution where the solution becomes fuzzier as time goes by. Resulting the fuzzy solution behaves slightly different from the crisp solution. There are theoretical difficulties that make the problem difficult where some states that large classes of two-point FBVPs has no solution under concept of Hukuhara differentiability (Bede, 2006). To overcome this shortcoming, concept of generalized differentiability has been introduced (Bede & Gal, 2004). There are many attempts have been proposed to solve FBVPs such as shooting-Adomian decomposition method (ADM) (S-ADM), shooting-secant (S-S) method and shooting-linearization (S-L) method (Bede & Rudas (2012); Fadhel & Al-Saedy (2009); Jamshidi & Avazpour (2012)). However, there is no study about the combination of shooting method and Euler method for solving FBVPs. Thus, this study will propose an alternative method on solving FBVPs called shooting-Euler (S-E) method under concept of generalized differentiability. Along these lines, Euler method will act as a numerical method which will approximate the solution of FBVPs in addition appraise the accuracy of the method.

1.5 Objectives

There are several objectives of this study:

(i) To study two-point FBVPs using the concept of generalized differentiability.

(ii) To solve two-point FBVPs numerically by using S-E method.

(iii) To compare the results between S-E method and S-ADM method of the two-point FBVPs

1.6 Scopes

In this study, the work focused on the understanding about concept of generalized differentiability and how a generalized differentiability is employed into two-point FBVPs. An example of linear two-point FBVPs is selected undergo investigation with Dirichlet boundary condition. This linear two-point FBVPs is solved by execution of shooting method with integration of Euler method in order to find the approximate solution. Above all, MATLAB is used to provide the approximate and graphical solution of the problem given. Lastly, the approximation solutions are compared with analytical solution provided.

1.7 Significant of the research

The finding of this study will redound to the benefit for the society by considering mathematics plays important role in sciences and technologies. Thus in this research, shooting method is used to solve two-point FBVPs under concept of generalized differentiability. From this study, it is important to understand the behavior on how shooting method under concept of generalized differentiability solves the FBVPs numerically. Furthermore, the shooting method with integration of Euler method will produce approximate solutions of the FBVPs given. By comparing the approximation solution with analytical solution, we can identify either the selected method is applicable to solve two-point FBVPs or not.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The purpose of this chapter is to summarize the existing research on solving FBVPs. This literature review subdivides into three main topics. In Section 2.2, the discussion will be prior works on FBVPs and fuzzy derivatives concept. In Section 2.3, follow by type numerical methods used in solving FBVPs. Finally in Section 2.4 is the overall conclusion of Chapter 2.

2.2 Fuzzy boundary value problems

In recent year, research on solving FDEs using numerical methods is growing tremendously. This is due to the ability of FDEs to mimic model of the dynamical system naturally under uncertainty environment. Chang and Zadeh (1972) claimed the first concept of fuzzy derivatives is introduced by Zadeh in 1965. Then, Zadeh's extension principle came after it (Dubois & Prade, 1982).

There are several problems in numerous fields looking for the solution of FBVPs. The fact is FBVPs has its own approach. FBVPs have two different approaches. According to Buckley and Feuring (2000), the first approach showed result solving the problems consist of fuzzy boundary values will give the obtaining solution still in fuzzy function. This is due to the derivatives that has been used in the differential equation is one of the fuzzy derivatives function family. The fuzzy derivatives function involved either concept of Hukuhara derivatives or any other in the generalized derivatives. Meanwhile, for the second approach is based on generating the fuzzy solution from the crisp solution (Buckley & Feuring, 2000).

Along the line, fuzzy initial value problems (FIVPs) also have its own way to solve the corresponding problem. It can be divided into three ways. Buckley and Feuring (2001) mentioned that the first approach by solving the crisp problem using extension principle. At this stage, the initial value provided is treated as a real constant. Later on, the real constant is replaced with initial fuzzy value. The second approach is using the concept of differential inclusion (Kaleva, 1987). This method takes α-cut of the initial value and converts differential equation into differential inclusion (Hüllermeier, 1997). The last approach is by assuming the derivatives being used either concept of Hukuhara derivative or in strongly Hukuhara derivative form.

The studies of FDEs really help in develop an advisable environment for the mathematical modeling of real world problems in which uncertainty or fuzziness occurs. Several approaches are advised to study about FDEs (Buckley & Feuring, 2000). Prior, the first approach is using the concept of Hukuhara differentiability for fuzzy valued functions. A study is conducted on the existence and uniqueness solution of FBVPs by using the concept of Hukuhara differentiability (Kaleva, 1987). According to Diamond and Kloeden (1994) conversely, this approach has shortcoming as the solution obtained becomes fuzzier as time goes by. This nature caused fuzzy solution behaves slightly different from the crisp solution. One of the foremost applications of fuzzy number arithmetic is handling problems regarding on FDEs and FBVPs (Buckley & Feuring, 2000).

In order to overcome the stated limitations, a concept of generalized differentiability is introduced (Bede & Gal, 2004). Apparently, the concept of generalized differentiability has shortcoming compared to the concept of Hukuhara differentiability where it leads FDEs has no unique solution (Bede & Gal, 2005). However, this disadvantage can be converted as an advantage where it enables the user to choose the singular points. This singular point will lead the changes on monotonicity of the solutions. Bede and Gal (2005) claimed the obtaining solutions are reversible (possibilistic term), stable and almost periodic solutions. The concept of generalized differentiability has extra advantages with respect to differential inclusions where it is more practical for numerical computation.

Researchers on FDEs under concept of Hukuhara differentiability and generalized differentiability are continued. Furthermore, several published results reinvestigate back. Research is conducted to investigate the equivalence between a two-point FBVPs and a fuzzy integral equation by using Green's function (O'regan, Lakshmikantham and Nieto, 2003). Yet, some example tested shows that two-point FBVPs for FDEs are not equivalent to the integral equation under concept of Hukuhara differentiability. According to Bede (2006), not all two-points FBVPs have solution under concept of Hukuhara differentiability.

Another investigation is made on the solution of two-point FBVPs by using the lateral type concept of Hukuhara derivative. This study tried to reduce four generated solutions into two different solutions as if there is monotone function exists in the two-point FBVPs (Liu, 2011). Two-point FBVPs divided into two different cases namely positive constant coefficients and negative constant coefficients. The outcomes for both cases are compared with properties of valid fuzzy level set and shape of boundaries. Liu (2011) concluded both cases give a fuzzy solution when boundary conditions are given as symmetric triangle fuzzy numbers. However, some solutions are not a valid fuzzy level set.

A study has been conducted on the class of BVPs for first order linear FDEs with impulses under the concept of Hukuhara differentiability. In order to solve impulse linear FDEs, Green's function is needed. This study concerns about existence and uniqueness solution of fuzzy boundary problem. Nieto and Rodriguez-Lopez (2013) solved the impulsive linear FDEs explicitly focusing on the boundary value conditions. Besides, the aim of this study is to develop some concrete result on the uniqueness and existence concept of Hukuhara differentiability solution toward BVPs for functional FDEs. In this case, Green's function act as a tool to extend the parallel analysis and aid results on the existence and uniqueness of Hukuhara differentiable solution to BVPs for first order impulsive linear FDEs (Nieto & Rodriguez-Lopez, 2013). As a conclusion, this paper found there are existence and uniqueness solution for the class of functional FDEs.

Recently, one study on existence, uniqueness, and characterization theorems for nonlinear fuzzy integrodifferential equations (IDEs) of Volterra type has been conducted. In this study, the existence, uniqueness, and other properties of nonlinear fuzzy IDEs are investigated under strongly generalized differentiability. Anyhow found that by using the strongly generalized differentiability, the fuzzy IDE has two local solutions (Abu Arqub, Momani, Al-Mezel, & Kutbi, 2015). According to Abu Arqub et al. (2015), the existence and uniqueness of two solutions of fuzzy IDEs of Volterra type are proved based on the Hausdorff distance under the assumption of strongly generalized differentiability for the fuzzy valued mappings of a real variable whose values are normal, convex, upper semi continuous, and compactly supported fuzzy sets in $\mathbb{R}$. In addition, an efficient computational algorithm is provided to guarantee the procedure and to confirm the performance of the proposed approach.

2.3 Numerical methods for solving two-point fuzzy boundary value problems

There are several numerical methods used to solve two-point BVPs such as finite difference method, shooting method and finite element through projection method (Kumar, Paul & Hoque, 2014). Ever since, shooting method became popular method to solve problems regarding on two-point BVPs. Shooting method known as an iterative method which is trying to identify the appropriate initial conditions related to initial boundary conditions which will provide the solution for the original BVPs (Meade, Haran & White, 1996).

A study is carried out on a numerical solution of BVPs for a second order FDEs. In this study, two-point BVPs are considered for a second order FDEs (Fatullayev, Can & Koroglu, 2013). These FDEs are expressed into four different systems of crisp BVPs under the concept of Hukuhara differentiability. Meanwhile, the numerical solution is obtained by implementation of shooting method. The results show this numerical method is effective in solving the problem given. Furthermore, this numerical method is significant to solve the nonlinear problem on the existence and uniqueness of the solution.

A new numerical method is introduced namely variational iteration method (VIM). According to Khader (2013), this method is known as a powerful mathematical tool for linear and nonlinear problems. Later on, VIM is used to solve problems regarding on two-point FBVPs for a second order FDEs under the concept of Hukuhara differentiability (Armand & Gouyandeh, 2013). The results are evaluated by comparing between exact solutions and approximation solutions. The comparison result revealed variational iteration method provides good approximation results toward the exact solutions.

Moreover, another study on solving two-point FBVPs using successive over relaxation (SOR) method is conducted. In this study firstly, the two-point FBVPs under the concept of Seikkala derivatives is undergone discretization via second order finite difference scheme to generate fuzzy linear system (Dahalan, Muthuvalu & Sulaiman, 2013). According to Seikkala (1987), Seikkala derivative is an extension of Hukuhara derivative and the fuzzy integral. For the fuzzy linear system, the numerical result is described through the implementation of SOR and Gauss-Seidel method (GS). Furthermore, the effectiveness of these two iterative methods is being compared. From the numerical perspective, it stated that SOR method is more advance regarding on the number of iterations and execution time as compared to the GS method. Plus in term of the accuracy, SOR method provides more accurate value of the Hausdorff distance compare to GS method. As a conclusion for the study, SOR method gives good approximation toward exact solutions than GS method.

Optimal homotopy asymptotic method (OHAM) is proposed to solve two-point FBVPs. According to Almousa and Ismail (2013), OHAM is one of the semi-analytical methods. The outcomes of the study showed OHAM is applicable to solve linear and nonlinear higher order of two-point FBVPs. In this study, OHAM provides a suitable way of controlling the convergence. This is due to the ability of the user to adjust the preferable convergence regions. Even by using low order approximation with only limited terms OHAM still gives accurate result yet solution converges to the exact