One Dimensional Heat Transfer through Rectangular Fin Using Finite Element Method (FEM)

by

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LIST OF ABBREVIATIONS

FEM	Finite Element Method
DQM	Differential Quadrature Method
OFEM	Optimum Finite Element Method
CFEM	Convection Finite Element Method

LIST OF SYMBOLS

T	Temperature
T_o	Initial temperature at $x = 0$
T_∞	Temperature around the surrounding
k	Thermal conductivity
q	Heat flux
h	Convective heat transfer coefficient
L	Length along x-axis
w	Width
t	Thickness
N	Number of elements
Z	Number of nodes
Q	Heat source
l_e	Length of an element
$\mathbf{r}_Q$	Heat rate vector
k_t	Element matrices

Pemindahan Haba Dalam 1 Dimensi Melalui Sirip Empat Segi Menggunakan Kaedah Unsur Terhingga

ABSTRAK

Disertasi ini membincangkan tentang pemindahan haba melalui sirip empat segi menggunakan kaedah unsur terhingga dalam satu dimensi. Dalam menyelesaikan masalah berkaitan pemindahan haba, pelbagai kaedah sedia ada boleh digunakan. Kajian ini akan memberi fokus kepada bidang ini dengan matlamat dan objektif untuk menyelesaikan masalah berkaitan pemindahan haba dalam satu dimensi dengan membina satu algoritma kaedah unsur terhingga dalam program MATLAB. Selepas itu, hasil yang diperoleh akan dibandingkan antara keputusan berangka dan pengiraan tepat. Dari sini, pembolehubah yang berbeza akan dicuba untuk menyiasat kesannya ke atas penyelesaian yang terdahulu. Pembolehubah yang ditukar adalah nombor nod, panjang sirip dan sifat konduktiviti bahan. Kajian ini akan mengaplikasi teknik penyelesaian berangka menggunakan kaedah unsur terhingga dalam proses membina satu program yang sesuai unutuk tujuan pengkomputeran. Hasilnya seperti yang dijangka, adalah selari dengan teori asal dengan perbezaan yang boleh diabaikan. Secara keseluruhannya, keputusan disertasi ini menunjukkan bahawa nombor nod memberi kesan ke atas ketepatan penyelesaian dengan perbezaan yang sangat kecil dan dianggap boleh diabaikan. Dalam waktu yang sama, panjang sirip adalah tidak berkaitan atau memberi kesan kepada nombor nod dan adalah pembolehubah yang tetap. Akhirnya, sifat konduktiviti bahan adalah satu-satunya pembolehubah yang dapat memberi kesan kepada nombor nod dan panjang sirip.

One Dimensional Heat Transfer through Rectangular Fin Using Finite Element Method

ABSTRACT

This dissertation present a study of heat transfer through rectangular fin by using finite element method. In solving heat related problem, there are many methods that can be used. This study will be focused on this area with the objectives to develop FEM algorithm using MATLAB for 1-D heat transfer problem. By applying this algorithm to solve 1-D heat transfer problem, we will compare both numerical and exact solution result. From here, the variables of different number of nodes, length and conductivity properties were tested to investigate the effect. By focusing on 1-Dimenesional heat equation, the Galerkin method is used. This study will apply numerical solution technique using FEM to solve 1-dimensional heat transfer through rectangular fins and developed a suitable algorithm for computational purpose. As expected, the results obtain are coherent with the theory with a negligible difference. Overall, this research shows that the number of nodes can affect the accuracy of result however the difference is so small that it does not have much effect. Meanwhile, the length does not relate or affect the number of nodes and are independent variable. Lastly, the material properties are the only variable that can affect both number of nodes and length.

CHAPTER 1

INTRODUCTION

1.1 Research Background

Heat is a form of energy that can cross the boundary of a two different system. When there are two different temperatures, the heat will flow from higher temperature to lower temperature. The movement of the heat is referred to as heat transfer. However, if a system is well insulated, there will be no heat transfer between it and its surrounding. Basically, there are three mechanisms of heat transfer. They are conduction, convection and radiation (Yunus et al., 2004).

In nowadays all over the world, heat transfer become one of the major study fields due to many real life problems arisen in engineering and scientific fields. In electrical appliances for example, the study of heat transfer become an important role. Most of the machines that deal with heat transfer use fin to accelerate the transferring process. If we know this heat transfer rate prior to its use, then the appliances may have higher safety and longer life time.

A fin in definition is an extended part of a body that increases the transferring rate of heat. In solving the heat equation, there are many numerical methods that have been introduced. These numerical solution techniques such as finite element method (FEM), differential quadrature method (DQM), finite difference method (FDM), boundary element method (BEM), runge-kutta method (RKM), raley-ritz method (RRM), and many more. Each has its own drawback in a way that it may either be too complex or require high computational effort which subsequently requires high cost.

Nevertheless, it is important to keep in mind that there are two basic techniques in solving engineering and scientific problems which mainly involve partial differential equations. The first technique is by using numerical method and the second technique is by using analytical method. Due to its suitability, numerical solution technique plays an important role in solving many engineering and scientific problems.

In this study however, we are interested to find one dimensional heat transfer through rectangular fin by using Finite Element Method (FEM). Finite element method (FEM) is one of the numerical solution techniques used to solve many thermodynamics problems in engineering and physical science including heat transfer problems. By using the selected technique, we will observe the result to find the effect of using different number of nodes in solution. We will also observe the effect of different fin length on temperature distribution.

1.2 Problem Statement

In solving heat related problem, there are many methods that can be used. However, in this study since we would like to investigate the effect of different number of nodes on different fin length, we will choose to use FEM. The result will be compared to with the exact result. Lastly, using the same problem, the different material properties will be used and the result will be investigated. When facing a dynamic problem, the first step is to reduce the problem into a set of ordinary differential equation in time by using discretization of FEM or nodal superposition approach which will be further discuss in Chapter 3. This is a well understood technique used widely in practice. Compared to the other method, FEM provides higher flexibility in modelling complex geometries thus were used to solve problems from various fields.

Equation 1 below shows the general 1-D heat equation with its respective boundary condition (Matthew J. Hancock, 2006):

$$\frac{d}{dx}\left(K\frac{dT}{dx}\right) + Q = 0 \qquad (1.1)$$

Where;

$T = T_o$ at $x = 0$

$Q = h(T_L - T_\infty)$ at $x = L$

k = thermal conductivity

q = heat flux = $-k\frac{dT}{dx}$

h = convective heat transfer coefficient

There are many numerical solution techniques available to solve this above equation. However, in this study we would like to investigate the efficiency and accuracy of the heat transfer in one dimensional rectangular fin using FEM. FEM provides higher flexibility in discretizing the element into sub elements. This is why FEM is chosen for this research.

For heat transfer problems, rapid changes of heat/temperature distributions take place near and at the fin boundary. It is very important to know this temperature changes behavior of the fin before choosing it. In conventional FEM, the element is usually subdivided into very small sub-elements uniformly which leads to huge amount of complexity which require large memory consumption and additional computation time. However, for the optimum(non uniform) FEM, it is only important to know the temperature changes only near and at the fin boundary.

Hence, instead of subdividing the element into very small sub-elements uniformly, it is better to subdivide the element into very small sub-elements at the boundary only, followed by relatively bigger elements toward the midpoint of the element non uniformly. This way any extra additional burden can be reduced. We can say that the heat transferred as above mentioned burden can be reduced using optimum heat transfer for non equal spacing.

Even so, the main focus in this study is to efficiently use the conventional FEM to find the solution for the stated problem. The element will be divided uniformly and we will use the different number of nodes to observe how the result changes. The problem will also be tested using different length chosen. Lastly, different material properties will be used and the step will be repeated to determine the effect it has on the solution. The results for both exact and numerical solution are recorded for study purposes. This study will reveal whether the changes applied will affect the solution and can be used as references for future research work.

1.3 Objectives

The objectives of this study are:

1. To develop FEM algorithm using MATLAB for 1-D heat transfer problem.
2. To apply this algorithm to solve 1-D heat transfer problem.
3. To find the numerical and exact result and compare the result.
4. To find the difference the effect of having different number of nodes and different fin length.
5. To apply the algorithm for the same problem using two different material properties.

1.4 Significance of study

Through this study, we are aimed to find the optimum heat transfer through the rectangular fin. This study is significant in finding the most efficient methods. Through the findings, the result can be applied to solve related problem in real life engineering and physical science problem. By completing this study, we will be able to:

1. Apply numerical solution technique using FEM to solve the heat transfer through rectangular fins.
2. Developed a suitable algorithm for computational purposes.
3. Compare the new result with the existing one.
4. Observe the result to see the effect of having different number of nodes and different fin length.

5. To apply the algorithm for the same problem to different material properties to see the pattern of the result.

1.5 Contribution of study

In this study, the problems stated earlier in Equation 1 will be solved independently using finite element methods (FEM). The result will be compared to the exact solutions or other published numerical solution if available. The result is expected to contribute a new idea to ignite a new development in computational mechanics. By meeting the objectives, the main contribution of this study can be listed as below:

1. Emphasized the significant of using Finite Element Method
2. FEM mesh points are determined accurately in terms of non-uniform discretization to obtain optimum numerical solution.
3. The performance of FEM optimal numerical results are evaluated comparing with some other published result for similar problem.

1.6 Conclusion

In conclusion, this study is conducted with the aim to find the optimum heat transfer through one dimensional rectangular fin by using finite element method (FEM) and compare this result with the exact result.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Heat is a branch of studies in thermodynamic mechanic. The ancient people regard that heat is much related to fire. However, it wasn't until much later that the theories regarding heat were introduces clearly. For more than a century after that the scientist and researcher have conducted a lot of studies regarding the subject. In modern world, the studies can be subdivided into three general areas as shown in Figure 2.1:

Figure 2.1: General areas of thermodynamic studies.

From the Figure 2.1 above, we can see that the foundation of the studies relies on theoretical part. In theoretical areas, the principles of mechanic and fundamental laws are studied. This field focuses on studying the theoretical part. The next step is the applied mechanic. Applied mechanics will transfers the theoretical knowledge into scientific and engineering applications, especially in regards of the construction of mathematical models of physical phenomena. With the knowledge in theoretical part, a model of real life phenomena problem can be solved. Lastly, the computational mechanics solves specific problems by simulation through numerical methods implemented on digital computers. In this research, the theoretical part will be used to solve the numerical solution. The equation will be applied to selected problem and will be used in computational part. The main focus is however relies on the computational part.

2.2 Heat Transfer

According to Yunus A. Cengel and Michael in their book "Thermodynamics an Engineering Approach" heat is defined as "the form of energy that is transferred between two systems (or a system and its surroundings) by virtue of a temperature difference" (Yunus et al., 2004). In other word, if the two systems are at the same temperature, there will be no heat transferred. An energy interaction can only be called heat if it takes place because of the existence of a temperature difference. Thus, heat is also known as the energy in transition. In thermodynamics, the word heat would simply refer to heat transfer. R. K. Rajput in his book "Engineering Thermodynamics" defines heat transfer as "the transmission of energy from one region to another as a result of temperature gradient". These heat transmissions can take place using three basic mechanisms (R. K Rajput, 2007):

1. Conduction: the transmission of heat in a substance from one part to another, or from one part to another part that has physical contact with it.
2. Convection: the heat transferred within a fluid by meaning of mixing one portion with another.

3. Radiation: the heat transferred throughout the space or matter other than the conduction or convection mechanism.

In real life situation, heat transmission can be achieved using one or combinations of these mechanisms. However, for a well insulated system with only a negligible amount of heat can pass through the boundary, there will be no heat transferred, Q = 0. This process is known as adiabatic process. This means that the heat will not pass through the insulated wall. The heat will be trapped in the system. This idea is the main factor behind the regular used thermos. In this study, we will observe the heat transfer in insulated tip rectangular fin (John et al., 2008: Heat and first law of thermodynamics, 2015).

2.3 Fin

Fin is an extended part of a body that increases the transferring rate of heat. These extensions can take variety of forms. The main purpose of a fin is to improve the convective removal of heat from a surface by putting an extension on that surface to increase its area. There are many types of fins such as rectangular, trapezoidal, triangular, concave parabolic and convex parabolic. In nature, the fin idea was observed from animal body. However, in today's world, the fin is used mostly in electrical and production of many products. For example, the fin is applied into compact laptop to reduce heat accumulation and increase the heat transfer rate from the device into surrounding. This will ultimately increase the efficiency and speed of the devices. In this study, we will use rectangular fin with insulated tip (Prabal Talukdar, Heat conduction through fins; Heat transfer from a fins, 2015).

2.4 Finite Element Method

The finite element method (FEM) is one of many numerical techniques that can be used for solving problems which are described by differential equations. It can also be used to formulate as functional minimization over the domain of a continuous physical system. The domain where the interest lied can be represented as a form accumulation or groups of finite elements. The domain of the solid structure was then divided into smaller part called sub-element. These elements form a relationship at a finite number of points on each element called nodes. This is how the model of the solid structure can be formed. Approximating functions in the finite elements are determined in terms of nodal values of a physical field which is sought. A continuous physical problem is transformed into a discretized finite element problem with unknown nodal values.

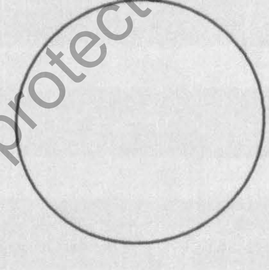

Figure 2.2: Two-Dimensional continuum Domain

Within the domain of each element a simple general solution is assumed to the governing equations. The specific solution for each element becomes a function of unknown solution values at the nodes. Applied to all nodes can result a finite set of algebraic equations to be solved for the unknown values (Tirupathi R C and Ashok D B, 1997).

In order to use FEM, there is certain consideration to take that is the background or situation in which FEM to be used and the main objective of the study in which it is involved the model simulation for physical system or approximation for numerical solution (Mathematics of finite element method, 2004).

2.4.1 FEM Working Procedure

To summarize in general terms how the finite element method works we list the main steps of the finite element solution procedure below (Tirupathi R C and Ashok D B, 1997; Strang G and Fix G J, 1973) .

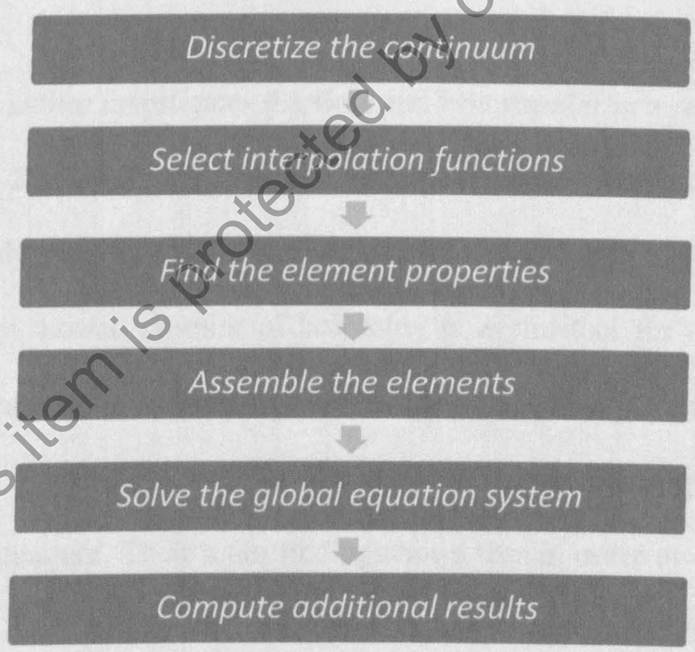

Figure 2.3: FEM solution procedure step

In the first step, the solution region was divided into finite element. Then we interpolate the element variable using interpolation function. The next step is to establish a matrix equation of the element which relates the nodal values of the unknown function to other parameters. The Galerkin methods are used as the variational approach. In order to find the solution for the general equation for the whole situation, all element equations are collected. In other word, local element equation for all element used for discretization are combined. Element connectivity's are used for assembly process. The boundary condition (if not accounted before in the element equation) should be imposed before finding the solution. In solving a global system equation, direct and iterative method can be used. As a result, the nodal values of the function tested can be known.

2.5 Reviews on other related work

2.5.1 Numerical Prediction of Flow and Heat Transfer in a Rectangular Channel

In this study, the author investigates the flow and heat transfer in a rectangular channel (S. Tiwari and G Biswas, 2003). The heat transfer in the duct was dictated by the flow structure, the study was directed towards characterization of the flow. At the end, the topological theory showed promise of becoming powerful tool for the study of flow structures. The limiting streamlines on the tube and the bottom plate revealed complex flow field. They compared their results with the well documented experimental results available in the literature. Their main finding shows that in order to describe the heat transfer behavior for the range of considered Reynolds numbers, there is no need to employ any turbulence model.