



**LMMSE-Based Low Complexity Image Denoising in
Non-Uniform Directional Filter Bank**

by

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A thesis submitted in fulfillment of the requirements for the degree of
Master of Science (Computer Engineering)

**School of Computer and Communication Engineering
UNIVERSITI MALAYSIA PERLIS**

2016

ACKNOWLEDGMENTS

All praises go to Allah who gives all favours. This work would not have been done without the support of my parents throughout. Thanks go also to my wife, the other half of me. Thanks go to my supervisor, Dr. Ruzelita binti Ngadiran, who was the main reason for my studying for a Master's, under her good supervision and commitment in my Final Year Project in my degree class. I thank her for her support and encouragement when I faced any difficulties in my study. I thank my co-supervisor, Prof. Puteh Saad, who drew the path to success in this research. I thank UniMAP for offering the Graduate Assistant scholarship that helped me complete my study. I thank all ENAC lab members, who were brothers in study time.

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Nyahhingar Imej Berasaskan Kaedah Kekompleksan Rendah LMMSE Menggunakan Penuras Miring Arah Tidak Seragam

ABSTRAK

Kajian ini bertujuan untuk mengurangkan atau membuang hingar dari imej dengan cara penggunaan pengubah yang tidak bertindan, berkesan, dan rendah kekompleksan. Nyahhingar imej adalah membuang hingar daripada imej untuk memastikan keaslian elemen asal imej dan menghapuskan segala penambahan yang tidak diinginkan. Kekompleksan nyahhingar berasaskan jelmaan bergantung kepada jelmaan/pengubah yang digunakan. Kajian terkini lebih menumpukan kepada meningkatkan prestasi nyahhingar tetapi mengabaikan kekompleksan jelmaan. Dalam kajian ini, kekompleksan jelmaan yang dikaji adalah jelmaan Contourlet (CT) yang cekap menangkap tekstur di dalam imej dan jelmaan penuras miring tidak seragam (NUDFB). Antara asas jelmaan yang sering digunakan dalam nyahhingar imej adalah jelmaan gelombang kompleks (CWT), Contourlet CT, jelmaan Contourlet tiada persampelan (NSCT). Prestasi kesemua jelmaan ini adalah baik tetapi ia kompleks dan mempunyai pembdehubah yang berlebihan. Hingar yang ditambah dalam imej ialah hingar Gaussian dan semua imej yang digunakan adalah imej skala kelabu. Kajian ini mencadangkan penggunaan Lelurus Minima Ralat Min Kuasa Dua (LMMSE) untuk nyahhingar dan NUDFB sebagai jelmaan asas. Kaedah yang dicadangkan menjelmakan imej menggunakan NUDFB kemudian mengenalpasti pekali untuk langkah seterusnya di dalam nyahhingar. LMMSE membandingkan pekali dalam bentuk resolusi yang berbeza-beza untuk menjangka hingar dan membuangnya. Dalam kajian ini, pengukuran ketepatan yang di gunakan ialah Nisbah Puncak Isyarat dan Hingar (PSNR) dan Kesamaan Struktur (SSIM). Imej yang terhasil daripada kaedah nyahhingar yang diusulkan menunjukkan penambahbaikan dalam keputusan. Perbezaan antara kaedah ambang dan kaedah yang diusulkan menggunakan PSNR adalah sebanyak 1 dB. Perbandingan dengan CT dan WT pula lebih baik terutama bagi imej dengan hingar yang berkadar tinggi. Dalam imej yang mempunyai informasi berarah seperti imej cap jari, kaedah yang diusulkan mempunyai nilai SSIM yang tinggi. Kaedah yang diusulkan mempunyai kekompleksan lebih rendah berbanding CT. Selain itu, ianya juga mempunyai prestasi yang lebih baik berbanding kaedah ambang keputusan yang lebih baik.

LMMSE-Based Low Complexity Image Denoising in Non-Uniform Directional Filter Bank

ABSTRACT

This dissertation is about creating a non-redundant, effective and low-complexity denoising method. Denoising an image involves removing noise from an image to keep the original elements of the image and remove unwanted additions. Transform-based denoising depends on the transform used in the denoising method. Recent works focus on improving the performance of the denoising, ignoring the complexity. This work studies the complexity of well-known transforms that capture texture in images sufficiently, known as Contourlet transform (CT) and non-uniform directional filter bank (NUDFB). Complex wavelet transform (CWT), CT and non-subsampled Contourlet transform are examples of the base of denoising methods in the majority of current work. All these transforms perform well but they are complex and involve redundancy. Noise applied to images in this work is Gaussian noise and all images used are greyscale; the Linear Minimum Mean Square Error (LMMSE) method was used for denoising, with NUDFB as the base transform. The method decomposes the image using NUDFB then recognizes the coefficients to the step of denoising. LMMSE compares the coefficients in different resolutions to predict and remove noise. For measuring work accuracy, Peak Signal to Noise Ratio (PSNR) and Structural Similarity (SSIM) are used. The resulting images from the proposed denoising method show improvement. The PSNR of the proposed method is higher than thresholding using NUDFB, about 1db. When comparing the proposed method with wavelet transform thresholding and CT, it has higher values than in CT and WT, especially in a high noise ratio. In images that contain directional information, such as fingerprint images, the proposed method has the highest SSIM. The proposed denoising method creates a way of denoising images using fewer requirements because of the low complexity. LMMSE also perform better than thresholding method.

CHAPTER 1

INTRODUCTION

1.1 Overview

In digital imaging, the image is represented as a matrix. Two types of accuracy limitations are blur and noise. Blurring is related to image acquisition systems, as the finite number of samples in the digital image must satisfy the conditions of Shannon-Nyquist sampling. The second disturbance is noise, caused by perturbation when receiving photons in image acquisition (Buades, Coll, and Morel 2005).

Denoising is a process of removing noise and keeping as much of the original data as possible. Different fields of image processing, such as image restoration, image segmentation, and many other fields, need an image close to the original. Image denoising is a lossy application, which means there is some loss of contained information. Noise is data added to the image, which needs to be eliminated. The challenge is how to estimate the image without noise, which is the denoising application. Denoising an image is the process of estimation of original image using a suitable model.

This dissertation describes development of a new denoising method with directionality for preserving texture in the image. This method is denoising based on linear mean square error (LMMSE) (El-Khamy et al. 2004). For directionality, a directional-based transform is used, a non-uniform directional filter bank (NUDFB) (Nguyen and Oraintara 2005). Based on a review of the best-known directional based transforms, NUDFB was chosen for many reasons. First, it is a transform that has less complexity and is an almost fully decimated transform (non-redundant). Secondly,

this transform contains directionality, which is required for capturing and preserving directional and texture information in the image. The third reason is that NUDFB has good results in non-linear approximation (NLA) which has a close results for some complex transforms such as Contourlet transform(CT) (Do and Vetterli 2005a). LMMSE was chosen as an approach to detect noise. It is a good detection method that will suit the functionality of the LMMSE. Arranging frequencies of NUDFB at each resolution level then applying LMMSE results in good noise detection. Improvement of the denoising results will be discussed in detail in the following chapters.

1.2 Background

Denoising images is an important process in image processing. Denoising using filters has been used by various researchers (Rajpoot and Butt 2012; Tsotsios and Petrou 2013). Filters were the basic way of denoising image until the invention of transforms such as wavelet transform and other types of denoising appeared. These methods concerned organizing the sub-bands of the wavelet transform to gather data in such way that noise will be detected. Wavelet transform is the base of many applications in image processing. It is a good transform for preserving singularities, but for the curves and smoothness of the image it has some weakness. CT is employed to overcome this problem, using the DFB which has good performance in high frequencies. This transform was a revolution in image processing, and researchers used it in many ways. The modified version of CT was non-subsampled CT (NSCT), which was the same as CT but without the down sampling process to give better transform, but with more complexity and more redundancy. These transforms are used for image denoising by two types of thresholding, hard and soft. The threshold process depends on the coefficients of the transform. The soft threshold has more focus for denoising of normal images. LMMSE was introduced

to provide denoising based on the structure of the image itself. LMMSE takes advantage of the transforms by using the transform sub-bands and comparing these for predicted noise. Complex transforms were used, such as over complete wavelet expansion (OWE) and NSCT. These transforms have high complexity and redundancy, and their use results in a complex and redundant denoising method.

1.3 Problem statement

Denoising methods proposed before LMMSE was used focused on the coefficients of the transform, such as hard thresholding and shrinkage. These methods remove noise in an efficient way but depend only on the coefficients of the transform used. A method that depends in the interscale of the transform itself was therefore needed, and the LMMSE met this need. The methods used were wavelet transform in OWE and NSCT, resulting in a better denoising method. Although the results were good for such transforms, they exhibit redundancy and have high calculation complexity. A method is therefore required that has directionality, multiresolution and less complexity and redundancy.

1.4 Objectives

The objectives of this work are:

- 1) To investigate the performance of multidirectional and multiresolution transforms.
- 2) To propose a new method of denoising using a combination of suitable directional transform for preserving texture in image.
- 3) To compare the performance of the denoising methods.

1.5 Scope

In this dissertation the image type used is greyscale. Noise applied to the images is Gaussian white noise, which represents many types of noise in the real noise applied to the image. The measurement used is PSNR and LMMSE. These two measurement methods are compare the original images with the predicted image after removing noise.

1.6 Outline

In this dissertation a denoising method that is low complex and non-redundant with texture preserving using directional property is proposed. In this chapter, the topic is introduced. Chapter 2 reviews the literature to explain related research and the history of the denoising of images, and some well-known transforms. Chapter 3 discusses the method in detail, and chapter 4 discusses the denoising results. The last chapter summarises the dissertation.

1.7 Motivation

Denoising is the process of removing noise and keeping as much original data as possible. Different fields of image processing, such as image restoration and image segmentation, need an estimated image close to the original. Image denoising is a lossy application, which means there is some loss of contained information. Noise itself is additional data which needs to be eliminated. The challenge is how to estimate the image without noise, while keeping the original data which is the core of the denoising application. Denoising an image is the process of estimating the original image using a suitable model. The key to a good denoising method is the property of detecting and estimating the original data. Texture is important data that needs a more special process to preserve it. This can be obtained by the directional filter bank (DFB).

Texture in an image has higher frequency, and as DFB gives better performance with high-frequency than with low-frequency data, it is preferred for better preserving of texture in the image. It is also necessary to use DFB to give directionality to the image. DFB filters the data in the image in multi phases, which results in better curve detection. For low-frequency data, it is possible to use other transforms that have good performance in preserving singular data. Preserving texture in an image is important and requires more processing, adding more complexity.

Less complex systems are better than higher complex ones that have the same functionality, because the latter require higher hardware specifications. Better performance will result from the greater of improved processes. The challenge is to have a less complex system with performance as good as that of higher-complexity ones.

CHAPTER 2

LITERATURE REVIEW

The denoising process is a very large area of image processing and the subject of many research. Noise that corrupts images may occur at any stage of image processing, due to the surrounding environment. Removing noise from images requires a technique that is sufficient to keep the original data. The core of most of denoising methods is how to predict the noise (E. Jebamalar Leavline. 2011). Non-transform based image denoising or special domain filtering, and transform-based techniques are the two types of image denoising. Both have the same general functions as shown in Figure 2.1.

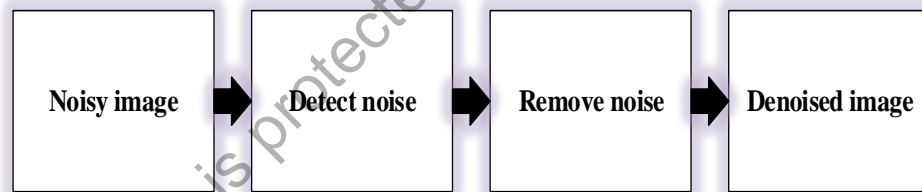


Figure 2.1: Denoising process

2.1 Non-Transform Based Denoising Methods

Denoising in a special domain may be categorized into linear and non-linear filtering. The Wiener filter is an example of the former, and the Median filter of the latter. These filters are suitable for overcoming pepper and salt noise (Jayasree and Kumar 2013). These types of denoising (Sasirekha and Thangavel 2014) are based on frequency filtering and familiarizing a cut-off frequency when noise is detected from the original data in the frequency domain. Their performance needs more time and depends on the cut-off frequency of the filter used. Moreover, these methods may cause artificial

frequencies in the denoised image. In general non-transform based denoising methods result in a blurred denoised image (Lopera, A. F., Cardona, & Orozco .etl 2014).

2.2 Transform-Based Image Denoising Methods.

A transform-based image denoising method depends on the transform as a base for decomposing the image. Basically, it uses the transform as a base for the decomposition process, then applies the denoising core process inside on the coefficients of the transform; after this it reconstructs the image using the transform. Many types of transform can be used for denoising, such as the popular discrete wavelet transform. Good denoising depends on having the right transform. Transform used for decomposing image should meet the advantage of the denoising method used.

2.2.1 Denoising Process

The denoising process is the core of detecting and deleting noise. Detecting noise in most cases is done by a mathematical calculation then applied for an original-like image. Details vary according to the problem that needs a solution, such as:

Thresholding, nonlinear approximation and linear minimum mean square error.

2.2.1.1 Thresholding

The threshold separates the connected higher frequency from the darker ones, and the technique can be used to detect noise at the threshold. The value of threshold determines the results (Wählby 2010). The technique is used for both denoising and

segmentation. For denoising, thresholding is usually used with transform-based denoising. Thresholding in the process of predicting the noise in a function f_x as:

$$f_x = x + n \quad 2.1$$

Where x is the original signal and n is the noise applied to the signal; denoising concerns predicting the signal x , which will be denoted as \hat{x} :

$$\hat{x} = f_x - n \quad 2.2$$

In this case, knowing the value of n is very important so that the value of \hat{x} is the closest value to the original value of signal x . Thresholding uses the wavelet transform for predicting noise in the signal. There are two types of threshold, hard and soft.

- a) **Hard threshold** is setting the coefficients of the noise signal to zero so that they are below the threshold value, and reconstructing the denoised signal. Hard thresholding is good for edge detection applications but not for image denoising.
- b) **Soft threshold** is widely used for image denoising because of its smoothness. The wavelet transform (WT) method (Donoho and Johnstone 1995) is used in transform domain denoising, which will be explained in section 2.2.2.1.

2.2.1.2 Non-Linear Approximation

NLA is a method of measuring the performance of a system by comparing the original signal with that produced by the system. It uses numerical computation to calculate complicated functions (target functions), with an easier method called the approximant (Hong-jun et al. 2013). For digital systems, NLA can give a good results of any systems performance, which makes it popular in image processing. For this reason it is used in the current work.

2.2.1.3 Linear Minimum Mean Square Error (LMMSE)

The LMMSE is a denoising method that uses interscale dependencies to detect noise from the original information. This method shows improvement when used with several transforms, due to its ability to calibrate with the transform to estimate the denoised image. It is based on the transform ability of extracting data in the coefficients of the transform; it also depends on the interscale of the image coefficients. LMMSE is therefore a good denoising scheme to use with a transform. Om and Biswas (2014) used WT with LMMSE to denoise 3D images. Their work proposed a development of the threshold method according to PSNR results. They used WT for denoising which is weak in preserving curves and smooth line. Fan Yang, (Ze Yu 2016) used LMMSE to denoise speckles in image with solving the over smooth the image. Improving algorithm was used in that work. The base transform was WT. They could improve the performance using the Non Local mean, but the texture and directional information is not well preserved. (Hosein M. Golshan and Reza P.R 2015) developed a denoising method based on LMMSE to denoise medical images. Magnetic resonance was the target to denoise. Patch-based L^2 was used to produce the patches samples to be used in the LMMSE to denoise the image. Improved results were presented in this work, but it

still needs more texture and direction preserving. Moreover, the similarity and dependency between patches or sub bands need to be stronger for better performance of LMMSE.

2.2.2 Transform Domain Denoising:

2.2.2.1 Wavelet transform (WT)

WT is a tool which divides a signal into multiple scales with different resolutions. This transform is mathematically designed for filtering data based on its vertical and horizontal dimensions, called two-dimension WT (2D WT). The foundation of WT is the square-integral function and group theory representation. A local representation is provided by WT in the time and frequency of an image. This makes it good for interscale dependencies of data in each scale (Mahela, Shaik, and Gupta 2015). 2D WT is flexible for denoising because of its frequency division. Figure 2.2 shows the spatial domain of WT. Figure 2.2 shows the flow of WT in filtering image. WT applied the synthesis filter to filter the low frequency data in image. For the high frequency data, analysis filter is applied. Both outputs of the analysis and synthesis filters are filtered by low and high filter again to output with four subband. First is low low, the first belongs to the x axis and the second for y axis in image, low high, high low and high high. Low low region is filtered again using the same way for more resolution levels.

The image is filtered by high and low filters. Greater resolution results from are petition of this process at each resolution. The detection of noise is more effective with more divisions of data in the image. This transform is used widely for denoising(Jain and Tyagi 2015)and the locally adaptive patch-based (LAPD) method is used for determining

the threshold after decomposing the image. Jain and Tyagi used it to preserve edges when denoising the image using WT.

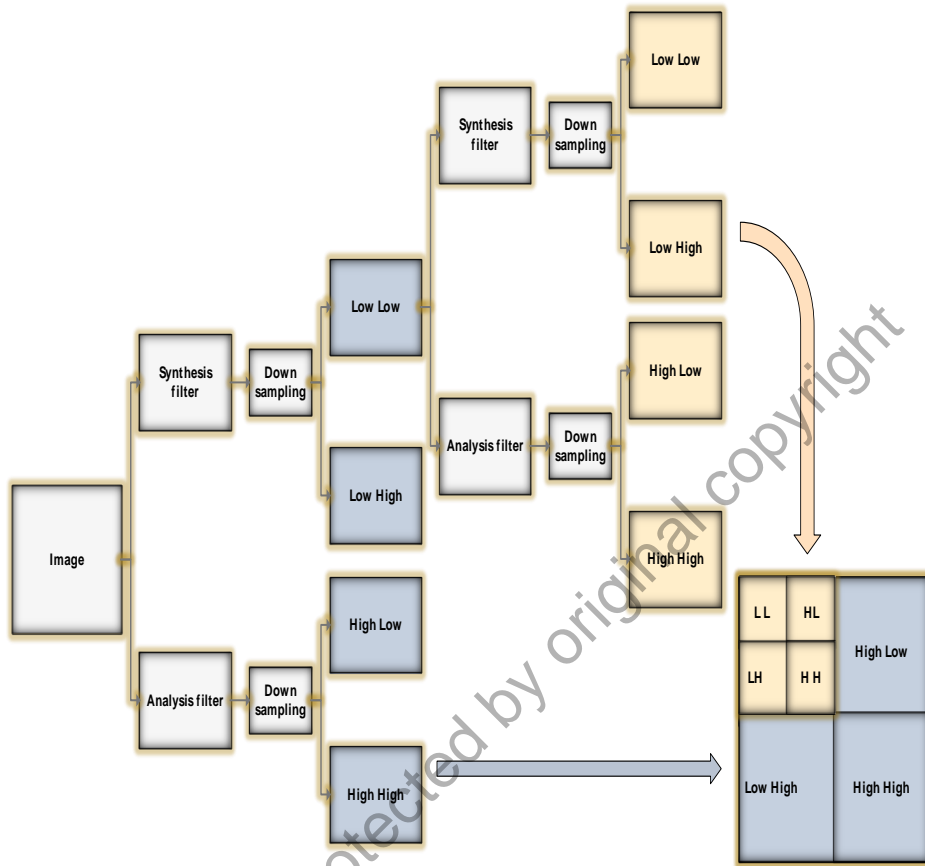


Figure 2.2: 2D WT spatial domain

This method is good for detecting noise and takes advantage of the strong interscale dependency in the wavelet, but it has an overlap when thresholding some noise. It may involve some image coefficients in the denoising process. In other work (Parrilli et al. 2012) a denoising method for deleting white Gaussian noise is proposed. Parrilli used a probabilistic similarity measure for matching the blocks when a developed shrinkage looks for optimum LMMSE. The problem of this method is using many steps for denoising: block gathering and similarity measuring using blocks, then the wavelet followed by the shrinkage. Moreover, this method does not preserve much texture or

directional data. Thresholding is still a valid method for denoising, and Deng and Liu (2015) improved the threshold performance by improving the constant deviation of the reconstructed images. Their results show an improvement over the original threshold, but this method still needs to detect more noise that has been missed by the coefficients of the WT. Further improvement in thresholding was proposed by Om and Biswas (2013), who addressed the problem of deleting important coefficients during denoising. They developed a method to detect noise more efficiently than previous threshold methods by working on the neighbourhood coefficients of each window in each scale of the WT. This method shows improved performance over other threshold methods. However, Om and Biswas focused on preventing the important coefficients from being deleted, so there was some deterioration in texture and smooth lines. This method detects noise based on singularities, which means it does not preserve smooth and texture information, unlike methods with a directional property. Other researchers have attempted to preserve texture in images using WT (Naimi, Adamou-Mitiche, and Mitiche 2015). They used dual tree complex wavelet transform (DTCWT) for denoising, and the Wiener filter with the DTCWT for better noise detection. This method shows good denoising, but it still a highly complex technique with redundancy.

WT is widely used for many image processing applications, especially denoising. The coefficients of this transform make it suitable for such applications, as it becomes easier to separate noise from the original data. However, this transform preserves singularities and has a weakness in preserving texture and smooth lines (Do and Vetterli 2005a). For preserving these types of data, researchers have had to increase the complexity of the transform, resulting in redundancy which is not desirable in image processing. For this reason, CT became widely used for preserving such data in the image (Ali Hamdi 2012).

2.2.2.2 Contourlet transform (CT)

CT is a transform that combines two stages of image filtering, one for low frequency data and the other for high frequency. For the low frequency part, Laplacian pyramids (LP) are used, and a directional filter bank (DFB) for the high frequency part. LP is a multiresolution process for preserving low frequency data when DFB is responsible for the high frequency, although it produces complexity and redundancy. DFB has good performance for preserving data in the high frequency range but is weaker at low frequencies. From this, it can be concluded that in CT each part completes the other. CT has become a base for a wide range of image processing and applications such as denoising. Figure 2.3 shows the spatial domain of CT. CT filters image firstly by applying LP to output with two regions.

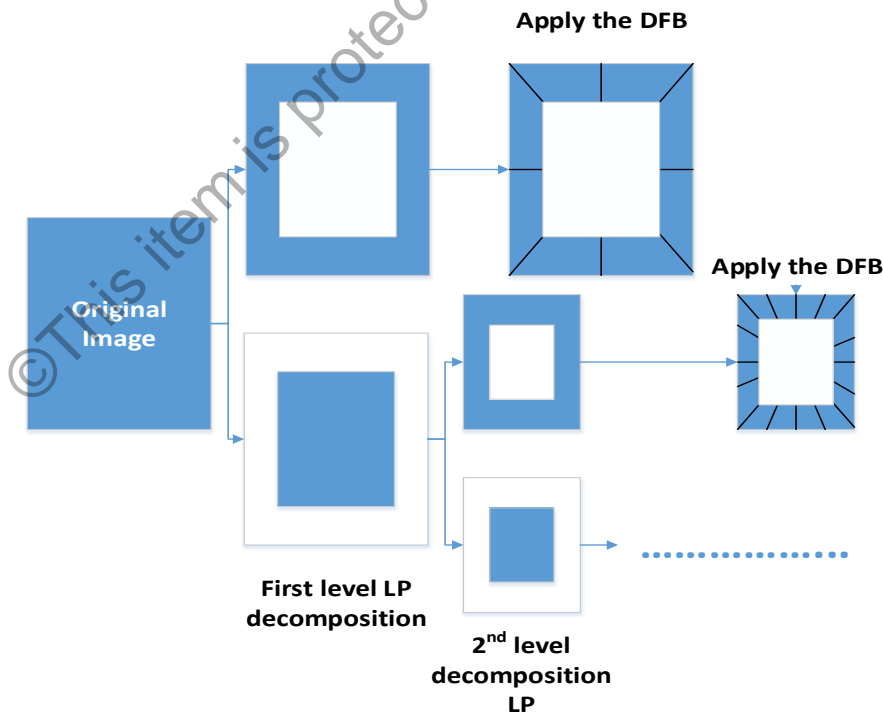


Figure 2.3: Contourlet transform special domain(Do and Vetterli 2005a)

Denoising using CT starts with thresholding. This method is used widely because of the coefficients extracted from CT. (Satheesh and Prasad(2011) developed the denoising of the image using the estimation of noise by mean absolute division. They then estimated the threshold for denoising. All this depends on the coefficients created by the CT. Their results shows improvement over the WT threshold method. However, their denoising results are not comparable with recent work, as the threshold alone will also erase some coefficients. Sun et al. (2013) used CT for removing the noise from medical images in a method called optical coherence tomography (OCT). These images have smooth lines that need to be preserved more accurately than WT is capable of. Non-Gaussian probability density function is applied to the decomposed coefficients produced by CT. A shrinkage is then used as the last step in denoising before reconstructing the image. In this method the denoising of the image performs better than in WT based on PSNR values. They estimate noise using algorithmic functions. Thresholding is then used to remove the noise from image. This method is good for multiplicative noise and but is very complex due to its use of CT. Other studies combine CT with other transforms for better denoising approaches. For example, Shang, Su, and Liu (2012) combined CT with kurtosis-based sparse coding (KSC). Combining CT with KSC increases the efficiency of determining the noise. KSC is a high-level statistical method that can estimate the image features' coefficients. Shrinkage is then used to remove noise. KSC-based denoising using the CT method improves the denoising of CT by overcoming the whole threshold denoising. Although it has good performance, it is still complex, redundant and has a weakness in determining noise in the low frequency band, since KSC was applied to the very high band produced from CT. In conclusion, using CT in denoising results in complexity and redundancy. (Song bo Wei, Jian Liu 2015) use CT to denoise image with avoiding removal of image coefficients by improving the universal thresholding and new

convergence factor. More coefficients are kept using this method. However the redundancy and complexity of this method is high which is undesired.

CT was designed to have greater ability to extract more coefficients by removing up and down sampling in the Contourlet to get the non-sub sampling Contourlet transform (NSCT) (da Cunha, Zhou, and Do 2006). As NSCT is CT with no subsampling, this makes its coefficients higher and better for applications like denoising. Denoising using NSCT is unsuitable for thresholding due to its very high complexity, and applying the threshold needs to go through many coefficients. Redundancy in NSCT is also much greater even than in CT. The inadequacy of NSCT for denoising forced researchers to look for a method other than thresholding. (Wang et al. 2013) denoised images using NSCT, but with the least square support vector machine (LS-SVM) for separating the noise from the original coefficients. This has advantages for directional frequency localization and fast implementation. However, it still has high complexity and redundancy. This transform is used widely with different denoising techniques (Li, Yin, and Fang 2012), (Liu et al. 2012), (Yin et al. 2013), (Shahdoosti and Khayat 2016) and (Argenti et al. 2009). Nevertheless all of these denoising methods have high complexity and redundancy regardless of their good denoising quality.

Using CT or NSCT help stop reserve texture and fine lines in images better than using WT, but they are complex and redundant systems. In spite of its low complexity and good performance, WT can only preserve singularities, which is a weakness compared with CT.

2.2.2.3 Non-uniform Directional Filter Bank (NUDFB)

Combining WT and DFB for a transform with low frequency and directionality is provided by NUDFB. WT has strength in preserving singularities gathered in the low