A RATIONAL CUBIC SPLINE TECHNIQUE FOR PRESERVING THE POSITIVITY OF DATA

by

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Suatu Kaedah Splin Kubik Nisbah bagi mengekalkan kepositifan data

ABSTRAK

Hasil kerja ini membincangkan masalah penggambaran lengkung (perwakilan data 2-dimensi) dan permukaan (perwakilan data 3-dimensi) dengan matlamat dan syarat yang rupa bentuk adalah licin dan boleh diubah. Untuk mencapai matlamat ini, kami mencadangkan satu interpolasi splin C^1. Bagi menyelesaikan masalah penggambaran data 2D, fungsi yang dicadangkan telah dibuat untuk mengandungi tiga parameter bentuk yang positif dalam setiap subselang dalam pembinaannya. Kekangan terhadap data diperoleh untuk satu parameter bentuk untuk memastikan pengekalan kepositifan melalui data positif yang diberi sementara baki dua parameter dibiarkan bebas untuk pilihan pereka untuk penghalusan lengkung dan/atau manipulasi. Interpolasi ini telah dilanjutkan kepada interpolasi splin dwi-kubik nisbah untuk menyelesaikan masalah penggambaran data 3D. Interpolasi lanjutan ini telah dihasilkan dan ia melibatkan enam parameter bentuk dalam setiap tampalan segiempat untuk pembinaan permukaan. Dalam kes ini, kekangan dicari untuk dua parameter bentuk untuk bagi mengekalkan bentuk positifnya manakala empat parameter yang lain adalah bebas mengikut keinginan pereka untuk pelicinan dan/atau manipulasi permukaan. Separuh daripada parameter bentuk ini disetkan pada arah x dan separuh lagi diset pada arah y supaya setiap satu daripada parameter bebas ini boleh diubah secara berasingan untuk mendapatkan model perwakilan data berasingan sewajarnya. Skim yang dibincangkan adalah efektif secara setempat terhadap selang data dan tidak dibenarkan dimasuki sebarang knot yang baru untuk mengekalkan kepositifan. Contoh berangka telah disediakan untuk menunjukkan bahawa skim yang dicadangkan telah berjaya dalam menghasilkan lengkung dan permukaan yang interaktif, licin dan boleh diubahsuai.

A Rational Cubic Spline Technique for Preserving the Positivity of Data

ABSTRACT

This work intends to address the problem of the visualization of curves (2-dimensional data representation) and surfaces (3-dimensional data representation) with the aim and provision that their display looks smooth and modifiable. In order to achieve these goals, we proposed a C^1 spline interpolation. For the treatment of the 2D data visualization problem, the proposed function has been made to contain three positive shape parameters in each subinterval of its construction. Simple data-dependent constraints are derived for single shape parameter to ensure preserving the positivity through given positive data while the remaining two parameters are left free for designer's choice for the curves' refinement and/or manipulation. This interpolation has been extended into rational bi-cubic spline interpolation to treat the problem of 3D data visualization. The extended interpolation has been made to involve six positive shape parameters in each rectangular patch of the surface construction. In this case, constraints are derived for two shape parameters for conserving the surface positivity while the remaining four parameters are left free according to designer's will for the surface smoothing and/or manipulating. Half of these shape parameters are set in the *x*-direction and the other half are set in the *y*-direction in such a way that each one of the free parameters can be changed separately to obtain different data representation models accordingly. The scheme under discussion is locally effective on the data intervals and does not allow to be inserted with any new knots to preserve the positivity. Numerical examples are provided to demonstrate that the proposed scheme is successfully producing interactive, smooth and modifiable curves and surfaces.

CHAPTER 1

INTRODUCTION

In many scientific fields such as Computer Graphics (CG), Geometric Modelling, Data Visualization (DV) and Shape Control (SC), parts of data representation models are required to be smooth and visually submissive under designer will, while preserving the inherited features of curves and surfaces in their displays. In this case, the designer needs to use some mathematical techniques or computer programs.

Since the mid of the last century, many methods intended to preserve the features of the curves and surfaces had been invented for the aim of satisfying many of engineering, medical, financial, geographic and many other scientific applications (Friendly, 2006).

Many methods with polynomials such as Lagrange approximation and spline that generated by using built-in MATLAB program are able to visualize 2-dimentional (2D) and 3-dimentional (3D) data representation but with the ability for showing unexpected undulation and for missing the characteristics of data visualization. Other mathematical data visualization techniques like cubic Hermite interpolation are able to remove the unexpected undulation but the features of the shape display model are not necessarily preserved.

The fundamental construction blocks of computer-based data visualization methods are curves and surfaces. Curves are used for representing 2D data visualizations while surfaces come in to represent 3D data.

This Chapter is organized as follows: Section 1.1 contains an introduction about data visualization and its applications. Section 1.2 talks about local and global interpolations. In section 1.3 and 1.4 we proposed an introduction about shape preserving and shape preserving interpolation respectively. In Section 1.5 we stated some basic definitions. Section 1.6 contains the study motivation. In section 1.7 we stated the problem of this research. Section 1.8 involves the objective of study. Scope of study is mentioned in section 1.9 and finally Section 1.10 contains thesis outlines.

1.1 Data Visualization and its Applications

Graphic depiction of quantitative information has deep historical roots. These roots belong to the histories of the earliest map-making and visual portrayal, and later belong to statistical graphics with applications and improvements in many fields of science and medicine that are frequently intertwined with each other. The roots of Graphic Depiction have also been joined with the rise of statistical thinking and extensive data collection for designing and commerce up through the 19^{th} century. Through history, a variety of developments contributed to the widespread use of data visualization nowadays. These include techniques for drawing and reproducing images, developments in mathematics and statistics, and new advancements in data collection, empirical observation and recording (Friendly, 2006).

In 1987, the National Science Foundation presented a report titled 'Visualization in Scientific Computing' which defined Data Visualization (DV) as a new application in the area of Computer Graphics and ensures the need to develop data visualization techniques (Bonneau, Nielson & Post, 2003).

Data Visualization was reported to be a method of computing which transforms the symbolic data into interactive geometric display, and in this case; designers can easily observe their simulations and computations as reality. Data visualization joins both image understanding and image composition altogether. The main goal of data visualization is to represent graphical information in effective and clear ways (Hansen & Chris, 2004). Large amount of data is transformed into graphs and figures either to complete underlying information from geography, meteorology, mining, basic sciences and medicine, or to communicate human imagination like aerospace industries, architecture, fine arts, advertising, and education by visualization tools (Abbas, 2012).

There are many requirements in Data Visualization; one is to preserve the shapes of inherited features to understand the meaning of underlying physical phenomena. The second requirement is shape control, i.e. the user can modify the shape of curves or surfaces without changing the original data. If the data visualization techniques do not fulfil these requirements then they would lead to the problem of shape preserving (Hussain, 2009).

Data Visualization techniques are very applicable and useful in numerous number of fields including the simulations of aerodynamics efficiency of ships, trains, automobiles and aeroplanes. Massive savings in both time and cost can be scored relying on mathematical models to implement computational visualization within aerodynamics assessment. Before that, the aerodynamics efficiency was evaluated by constructing small scale models and testing them in wind tunnels based on their designs. It had been a process that was very costly, time consuming process and subjected to errors due to the scaling factor (Hussain, 2009). Treinish (1995) applied visualization techniques to rainfall data as a function of elevation, geographical location. Data visualisation is very applicable in Computerized Numerical Control (CNC) machining (Poliakoff, Wong & Thomas, 1999) and 3D printers.

The basic construction blocks of computer-based data visualization techniques are curves and surfaces. Curves can be defined as locus of points that have only one degree of freedom. On the other hand, surfaces can be defined as locus of points where the degree of freedom is two. Visualizations of given sets of 2D data points are curves and visualization of given sets of 3D data points are surfaces (Abbas, 2012).

As far as problems of real life are concerned, the 3D form of data is the main and most common type to face. These can be further classified into Regular and Irregular (scattered) data types (Hussain, 2009). The first type can be defined over a rectangular grid. Alternately, the second type are the collinear data that can rather be defined over a triangular grid in comparison to the regular 3D data (Ramos & Enright, 2001).

1.2 Local and Global Interpolations

Interpolation is said to be local if the solutions are being found on the basis of information upcoming from a fixed and relatively small number of neighbouring data points, on the other hand, interpolation is said to be global if the solutions are being found on the basis of the solution of a large system of equations, often based on a minimisation problem (Kuijt, 1998).

1.3 Shape Preserving

Shape preserving is a problem that is a part of Data visualization. It means dealing with unexpected changes occurring in both 2D and 3D data representation models so as to preserve the properties of inherited shape, positivity, convexity and monotonicity. Shape preserving problem had been defined by Pan and Cheng (2012) as follows: “Preserving the geometric shape of the given data points”. To attain those goals, too many techniques of shape preserving had been developed by some researchers. Generally, the techniques of shape preserving may be classified into two types. The first kind is shape preserving techniques that are based on their control polygon or control mesh. The second kind is the shape preserving technique that is based on the interpolation points of knot points or fitting points. Examples of the first kind are: C-curves and surfaces, Bézier curves, B-spline curves and surfaces, and H-curves (Pan & Cheng, 2012). In this research we are going to focus on techniques of shape preserving interpolation.

1.4 Shape Preserving Interpolation

Karim, Pang and Saaban (2015) described the shape preserving interpolation as a process that involves some mathematical derivation to visualize 2D and 3D data sets in the form of piecewise curves and surfaces, respectively by implying some degree of smoothness.

A typical technique for curves begins with a set of 2D points and employs interpolation to compute a smooth curve that passes through 2D data (Salomon, 2007). In the same way, we can say that the typical technique for surfaces begins with a set of 3D data to compute a smooth surface that passes through 3D data.

Notably, the shape preserving interpolation is regarded to be successful if it was able to preserve original shape of data sets, for instance, if the given data are positive, then the final brought out curves and surfaces must be positive in each interval and patch and they should be visually pleasing for the computer graphics representation models (Karim et al., 2015).

1.5 Basic Definitions

In this Section we are going to introduce some basic definitions and terminology for the establishment of the concepts and the equations that are used in this research.

1.5.1 2 Dimensional Positive Functions

A set of 2D data $\{(x_i, f_i), i = 0,1,2,...,n\}$ with $x_0 < x_1 < ... < x_n$ can be defined as positive 2D data if

$$f_i > 0 \quad \text{or} \quad f_i \geq 0 \tag{1.1}$$

There exist some quantities that are always positive and their graphical displays are nonsense if they were negative, for instance, positive 2D data can be seen in levels of gas discharge in many chemical reactions (Sarfraz, 1993a) as well as in density and crops production as functions of time.

1.5.2 3 Dimensional Positive Functions

A set of 3D data $\{(x_i, y_j, F_{i,j}), i = 0,1,2,...,m, j = 0,1,2,...,n\}$ can be defined as positive 3D data if

$$F_{i,j} > 0 \text{ or } F_{i,j} \geq 0 \tag{1.2}$$

3D positive function can be appear in bivariate functions like size meteorological, monitoring maps, image processing and instance probability distributions (Abbas, Majid, Ali, & Sciences, 2012).

Positivity is one of the most significant features of curves and surfaces that make sense to study. Therefore, this research is going to address the problem of positivity preservation.

1.5.3 2 Dimensional Monotone Functions

According to Beliakov (2005), set of 2D data $\{(x_i, f_i), i = 0,1,2,...,n\}$ with $x_0 < x_1 < ... < x_n$ can be defined as monotonically increasing if:

$$f_i \leq f_{+1} \quad (1.3)$$

And monotonically decreasing if:

$$f_i \geq f_{+1} \quad (1.4)$$

Empirical option pricing models in finance are always monotone. Other examples of monotone 2D function is erythrocyte sedimentation rate (Butt, 1991).

1.5.4 3 Dimensional Monotone Functions

According to (Beliakov, 2005), a 3D data set $\{(x_i, y_j, F_{i,j}), i = 0,1,2,...,m, j = 0,1,2,...,n\}$ is said to monotone if:

$$\left.\begin{matrix} F_{i+1,j} > F_{i,j} \\ F_{i,j+1} > F_{i,j} \end{matrix}\right\} \quad (1.5)$$

Monotone 3D function can be found in blood uric acid level in those patients who are suffering from gout generated from stress and strain of a material. Other example of monotone function is work as a function of both force and displacement.

1.5.5 2 Dimensional Convex Functions

According to (Abbas, 2012), a set of 2D $\{(x_i, f_i), i = 0,1,2,...,n\}$ is called convex if:

$$\Delta_i < \Delta_{i+1}, \ i = 0,1,...,n-2 \quad (1.6)$$

Conversely, a 2D function is called concave if:

$$\Delta_i > \Delta_{i+1},\ i = 0,1,...,n-2 \tag{1.7}$$

Where

$$\Delta_i = \frac{f_{i+1} - f_i}{h_i},\ h_i = x_{i+1} - x_i,\ i = 0,1,...,n-1$$

Convex data plays an important role in various scientific applications such as telecommunication system designs, optimisation, and approximation theory (Sarfraz, Hussain & Hussain, 2012). Additionally, convexity happens in nonlinear programming for instance, parameter estimation, and measuring the sensitivity of time duration it takes by a bond to change with respect to respective rates of interest (Karim, Hasan, & Sulaiman, 2014).

1.5.6 3 Dimensional Convex Functions

According to (Abbas, 2012), a given set $\{(x_i, y_j, F_{i,j}), i = 0,1,2,...,m, j = 0,1,2,...,n\}$ of 3D data is said to be convex if:

$$\left.\begin{array}{l} F_{i,j}^{x} < \Delta_{i,j} < F_{i+1,j}^{x} \\ F_{i,j}^{y} < \hat{\Delta}_{i,j} < F_{i,j+1}^{y} \end{array}\right\} \tag{1.8}$$

where

$$\Delta_{i,j} = \frac{F_{i+1,j} - F_{i,j}}{h_i}, \ \hat{\Delta}_{i,j} = \frac{F_{i,j+1} - F_{i,j}}{\hat{h}_j}, \ \hat{h}_j = y_{j+1} - y_j$$

$F_{i,j}^x$, $F_{i,j}^y$ denote the values of the given knots that are used for the smoothness of interpolant.

3D convexity can be seen in designing well shaped smooth surfaces arise in manufacturing the TV-screens. In order to accomplish with the demands of the customer, as flat as possible TV-screens are most appreciated. In the surface designing sense we can say that the screens must preserve the convexity (Kuijt, 1998).

1.5.7 Continuously Differentiable Functions

A function f is said to be continuous of the first degree (C^1) if all the single partial differentials exist and are continuous (Mukherjee, 2015).

In general, a function f is said to be continuous of degree n (C^n) on the interval $[a,b]$ if :

1. $f(x)$ is n times differentiable on $[a,b]$.

2. If $f^n(x)$ is itself continuous on $[a,b]$.

1.6 Motivation of Study

There exists many 2D and 3D data generated either by physical phenomena or by mathematical functions. These data have specific properties, for instance the data generated by the function:

$$F(x,y) = (x^2 - y^2)^2 + 1\ ,\ -3 < x, y < 3 \qquad (1.9)$$

These data as stated in Table 1.2 are always positive, i.e. locate over the plane $Z = 0$.

According to Gregory (1986), Gregory and Sarfraz (1990), Butt and Brodlie (1993); Brodlie, Mashwama and Butt (1995), Duan, Xu, Liu, Wang and Cheng (1999), Costantini and Manni (2003), The schemes used to visualize such cases are required to be achieved with these prominent points:

- Preserve the inherited positivity of 2D data representation (curves) and 3D data representation (surfaces).
- Visualize the given data in the form of curves and surfaces with analytical or geometrical continuity of some order in such a way the data representation looks smoother than the normal data representations.
- Provide ability to refine and manipulate the resulting curves and surfaces.

Recently, a number of mathematical techniques, like spline interpolation and Hermite interpolation, these techniques have been developed for the treatment of these requirements, but each one of them shows some shortcomings. The following examples elaborate these shortcomings.

Example 1.1:

Table 1.1: Positive 2D data.

i	1	2	3	4	5	6	7
x_i	2	3	7	8	9	13	14
f_i	10	2	3	7	2	3	10

Table 1.1 contains a set of 2D positive data that will be plotted using some mathematical techniques without any constrains to preserve positivity through representation of positive data.

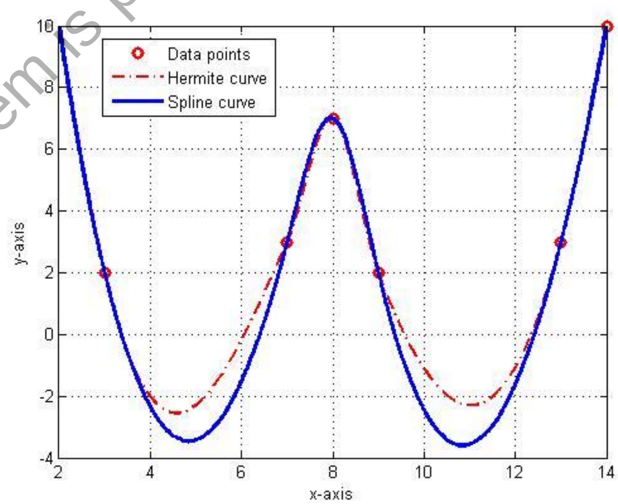

Figure 1.1: Plot of data in Table 1.1 using cubic Hermite interpolation and using Spline (built-in-MATLAB function)