MAGNETOHYDRODYNAMIC STAGNATION POINT FLOW AND HEAT TRANSFER TOWARDS A STRETCHING SHEET WITH SUCTION IN A NANOFLUID

by

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TABLE OF CONTENTS

	PAGE
THESIS DECLARATION	i
ACKNOWLEDGEMENT	ii
TABLE OF CONTENTS	iii
LIST OF TABLES	vi
LIST OF FIGURES	vii
LIST OF SYMBOLS	viii
ABSTRAK	xi
ABSTRACT	xii
CHAPTER 1 INTRODUCTION	
1.1 General Information	1
1.2 Objective, Scope and Research Methodology	2
1.3 Thesis Outline	3
1.4 Heat Transfer	4
1.5 Boundary Layer Theory	5
1.6 Suction Effect	5
1.7 Important Parameters	6
1.7.1 Prandtl number, Pr	7
1.7.2 Reynolds number, Re	7
1.7.3 Local skin friction coefficient, Cf	8

1.6.4 Nusselt number, Nu 9

1.6.5 Sherwood number, Sh 10

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction 11

2.2 Buongiorno Model 12

2.3 Tiwari and Das Model 12

2.4 Propose Problem: Stagnation Point Flow and Heat Transfer towards a Stretching Sheet with Suction in a Nanofluid 13

2.2 Shooting Method 16

CHAPTER 3 GOVERNING EQUATIONS

3.1 Governing Equations 19

3.2 Physical Model and Boundary Conditions 21

3.3 Similarity Transformations 22

3.4 Solution Procedure using Shooting Method 33

CHAPTER 4 NUMERICAL RESULTS AND DISCUSSION

4.1 Introduction 37

4.2 Result and Discussion on Heat Transfer Characteristics 37

4.3 Result and Discussion on Velocity, Temperature, and Nanoparticles Fraction Profiles 42

CHAPTER 5 CONCLUSIONS AND FUTURE RESEARCH

5.1 Conclusion 46

5.2 Summary of Research 47

5.3 Suggestion for Future Research 48

REFERENCES 50

APPENDIX A 56

LIST OF TABLES

NO.		**PAGE**
Table 4.1	Comparison of values of $f''(0)$ with previous results when the effects of $M=0$, $Nb=0$, $Nt=0$ and $S=0$	38
Table 4.2	Comparison results for $-\theta'(0)$ with previous results effects of $M=0$, $Nb=0$, $Nt=0$ and $S=0$	39

LIST OF FIGURES

NO.		PAGE
Figure 3.1	Physical model and coordinate system.	22
Figure 4.1	Variation of the skin friction coefficient with M for different values of S when $Le = 2$, $Nt = 0.1$, $Nb = 0.1$ and $\Pr = 1$	40
Figure 4.2	Variation of the local Nusselt number with M for different values of S when $Le = 2$, $Nt = 0.1$, $Nb = 0.1$ and $\Pr = 1$.	40
Figure 4.3	Variation of the local Sherwood number with M for different values of S when $Le = 2$, $Nt = 0.1$, $Nb = 0.1$ and $\Pr = 1$.	41
Figure 4.4	The effect of suction parameter S on velocity profile $f'(\eta)$ when $Nt = 0.1$, $Nb = 0.1$, $Le = 2$, $A = 0.1$, $M = 1$ and $\Pr = 1$.	43
Figure 4.5	The effect of suction parameter S on temperature profile $\theta(\eta)$ when $Nt = 0.1$, $Nb = 0.1$, $Le = 2$, $A = 0.1$, $M = 1$ and $\Pr = 1$.	43
Figure 4.6	The effect of suction parameter S on nanoparticle concentration $\phi(\eta)$ when $Nt = 0.1$, $Nb = 0.1$, $Le = 2$, $A = 0.1$, $M = 1$ and $\Pr = 1$.	44

LIST OF SYMBOLS

Nomenclature

A	velocity ratio parameter
B_0	magnetic field strength
C_f	skin friction coefficient
C	concentration
C_w	concentration at the surface of the sheet
C_∞	ambient concentration
D_B	Brownian diffusion coefficient
D_T	thermophoresis diffusion coefficient
f	dimensionless stream function
ϕ	dimensionless concentration function
k	thermal conductivity
Le	Lewis number
M	magnetic parameter
Nb	Brownian motion parameter
Nt	thermophoresis parameter
Nu_x	local Nusselt number
$\Pr$	Prandtl number
Re_x	local Reynolds number
T	temperature of the fluid

Sh_x	local Sherwood number
T_w	temperature at the surface of the sheet
T_∞	ambient temperature
U_∞	free stream velocity
u, v	velocity component along *x*- and *y*-direction

Greek symbols

η	dimensionless similarity variable
μ	dynamic viscosity of the fluid
υ	kinematic viscosity of the fluid
$(\rho)_f$	density of the basefluid
$(\rho c)_f$	heat capacity of the base fluid
$(\rho c)_p$	effective heat capacity of a nanoparticle
ψ	stream function
α	thermal diffusivity
σ	electrical conductivity
θ	dimensionless temperature
τ	parameter defined by $\dfrac{(\rho c)_p}{(\rho c)_f}$
τ_w	wall skin friction
q_w	wall heat flux
j_w	wall mass flux

Subscripts

∞	condition at the free stream
w	condition at the surface
f	base fluid
p	nanofluid

Aliran Titik Genangan Magnetohidrodinamik (MHD) dan Pemindahan Haba terhadap Permukaan Meregang dengan Sedutan dalam Nanobendalir

ABSTRAK

Aliran titik genangan magnetohidrodinamik (MHD) dan pemindahan haba terhadap permukaan meregang dalam nanobendalir dengan sedutan dikaji. Persamaan asas yang menakluk aliran bendalir dan pemindahan haba dijelmakan kepada persamaan pembezaan biasa tak linear menggunakan penjelmaan keserupaan, sebelum diselesaikan melalui kaedah tembakan. Kesan parameter sedutan ke atas pekali geseran kulit, nombor Nusselt setempat dan nombor Sherwood setempat serta profil-profil halaju dan suhu diperolehi dan dianalisis. Keputusan berangka menunjukkan bahawa magnitud pekali geseran kulit, nombor Nusselt setempat dan nombor Sherwood setempat meningkat dengan kesan sedutan.

Magnetohydrodynamic (MHD) Stagnation Point Flow and Heat Transfer towards a Stretching Sheet with Suction in a Nanofluid

ABSTRACT

The magnetohydrodynamic (MHD) stagnation point flow and heat transfer towards a stretching sheet in a nanofluid with suction is investigated. The basic equations governing the fluid flow and heat transfer are transformed into nonlinear ordinary differential equation using a similarity transformation, before being solved by means of a shooting method. The effects of suction parameter on the skin friction coefficient, the local Nusselt number and the local Sherwood number as well as the velocity and the temperature profiles are obtained and analyzed. The numerical results indicate that the magnitude skin friction coefficient, the local Nusselt number and the local Sherwood number increase with suction effect.

CHAPTER 1

INTRODUCTION

1.1 General Information

The term "nanofluid" was introduced by Choi and Eastman (1995) in describing the mixture of base fluids such as water, oil, ethylene glycol with nanoparticles. Nanofluid shows better stability with highly effective thermal conductivity. Many studies prove that the presence of nanoparticles in the base fluid increases the fluid thermal conductivity as well as increase the heat transfer characteristics. Base fluids provide very important industrial processes involving heat transfer. Choi et al. (2001) have reported that the thermal conductivity of the base fluid increases approximately two times with the addition of small amounts (less than 1% by volume) nanoparticles to the base fluid.

It is noted that Khan and Pop (2010) was the first published research paper on the boundary layer flow problem over a stretching sheet in a nanofluid. Makinde and Aziz (2011) have solved a similar problem under convective boundary conditions. Later, Hady et al. (2012a) investigated the flow field and heat transfer over non-linear stretching surface in a nanofluid with radiation effects. Nadeem and Lee (2012) investigate the problem of flow in nanofluid over an exponentially stretching surface. A good amount of literature on this type of fluid can be found in the book and research papers by Das et al. (2007), Kakaç and Pramuanjaroekij (2009), Wang and Mujumdar (2007), Daungthongsuk and Wongwises

(2007), Trisaksri and Wongwises (2007), Wong and Leon (2010), Saidur et al. (2011), Wen et al. (2011), Mahian et al. (2013) and many others.

1.2 Objective, Scope and Research Methodology

This study focus on the following aims:

1. To carry out the mathematical analysis of the problem of magnetohydrodynamic (MHD) stagnation point flow over permeable stretching sheet in a nanofluid with suction effect at the boundary.
2. To solve the problem of the MHD stagnation point flow over a permeable stretching sheet in a nanofluid.
3. To investigate the effects of suction on the velocity, temperature profile and nanoparticle volume fraction and heat transfer characteristics.

The scope of study is focused on the effect of suction on the MHD stagnation point flow towards a stretching sheet in nanofluid.

The research methodology of this research will be carried out in four stages (methods), as follows:

Level 1: Partial differential equation is transformed into ordinary difference equations using similarity transformations. The resulting outcome solution is called the similarity solution, which has features similar to the original equation solution.

Level 2: Ordinary differential equations are then solved using the shooting method.

Level 3: The results obtained are compared with the results reported by previous researchers for some special cases, to support the validity of the numerical results obtained.

Level 4: The influence of the involved parameters of the boundary layer flow and heat transfer characteristics will be discussed.

1.3 Thesis Outline

This project is divided into five important basic chapters including this introductory chapter. In this Chapter 1, introduction covers the terms, parameters, theories and the processes involved to introduce the area of study in more detail. Besides, the objectives and scope of the study are also presented in this chapter to further clarify the goals of the study.

Then, the review data taken from the previous works shall be discussed more in our own literature reviews in the Chapter 2, where it is very essential to determine the method of implementation and the general overview of the study for examples, the numerical study about the shooting method that will be used in this analysis, before discussing it in further detail in the next chapters.

In Chapter 3, the physical model and the boundary conditions used in the problem of the MHD stagnation point flow and heat transfer towards a stretching sheet in nanofluid with suction will be discussed. The governing equation of the present problem is shown and the similarity transformation used is also presented. Then, the solution procedure by using the shooting method is discussed.

Next, the numerical method used to solve the research problem will be discussed in Chapter 4. The numerical algorithm to solve the differential equation subject to associated boundary condition which can be programmed using Maple software is performed in this chapter. Also, discussions on relevant physical quantities such as the skin friction coefficient, local Nusselt number and Sherwood number are presented in the Results and Discussions subsections. Some discussions on temperature profiles, velocity profiles and nanoparticle concentration are also included. Our present result for a particular case has been compared with existing results from the literature for a similar problem in the stagnation point flow. The comparison shows a good agreement, and thus, we proceed to get the new results for different values of the suction parameter in the stagnation point flow. All related figures are presented, and some results are also given in the form of tables in this chapter. These data can be used by another researcher as reference for comparison in the future.

The final chapter, Chapter 5, include some concluding remarks, the summary of research and suggestions for future research. All the references in this thesis are listed in the reference section at the end of Chapter 5. The Appendix A represent the numerical algorithm used in the Maple software program.

1.4 Heat Transfer

Heat transfer is a discipline that involves the transfer of thermal energy from one physical system to another. Heat transfer mechanism is divided into conduction, convection and thermal radiation. In heat transfer, conduction is the transfer of thermal energy between regions due to the temperature gradients. Heat flows from a higher temperature area to a lower temperature region, thereby reducing the difference in temperature and reached thermal equilibrium. Convection is the movement of molecules in a fluid. It occurs because of the temperature difference between the fluid and the surface. It cannot occur in solids due to the fact no flow or diffusion can occur in it. Thermal radiation is considered one of the basic methods of heat transfer, even if it does not involve convective heat transport.

1.5 Boundary Layer Theory

Ludwig Prandtl was the person introduced the boundary layer theory in 1904 (Acheson, 2001). Boundary layer refers to a thin layer adjacent to the surface of the body, where the effects of viscosity cannot be neglected. The main idea presented by Prandtl is that the flow can be divided into two parts, inviscid flow in the main part (big part) and a thin layer adjacent to the surface of the body, known as the boundary layer. In thin layers, the frictional force must be considered while for the outside layer, the frictional force is very small and can be ignored (Schlichting, 1979). The boundary-layer theory is used in solving the problem of boundary layer flow and heat transfer (Bejan, 1984; Cebeci & Bradshaw, 1988). Governing equations for the fluid motion are the Navier-Stokes equations. These equations are the nonlinear partial differential equations which are hyperbolic or elliptic. Nonlinearity makes it difficult or impossible for the problem to be solved. By using the boundary layer approximation, the Navier-Stokes equations can be reduced to be parabolic partial differential equations, which are easier to solve.

1.6 Suction Effect

Recently, the effects of suction on moving plate or boundary layer have been investigated by many researchers in fluid field. Hatami and Ganji (2013) investigated heat transfer and nanofluid flow in suction and blowing process between parallel disks in presence of variable magnetic field. The suction can alter the flow field; hence can be changed on skin friction and heat transfer coefficient. For suction increase, the skin friction coefficient increase and heat transfer also increase, due to the velocity profile increase. Bachok et al. (2011) reported about the boundary layer flow over a moving surface in a nanofluid with suction or injection. They found that, the suction will increase the velocity profile and nanoparticle concentration however opposite trends where it could be reduce the temperature profile. Hady et al. (2012b) studied about the effect of suction/injection on natural convective boundary-layer flow of a nanofluid past a vertical porous plate through a porous medium. The suction effect decreases the thermal boundary layer thickness. They stated the motion of nanoparticles in the nanofluid cause the nanoparticles close together and create the forces such as the Van Der Waals force and nanoparticles become agglomerate each other. For this reason, the surface area of volume ratio of nanoparticles varies the wall layer of the plate; the thickness in this layer increase thus reduced the thermal conductivity of the fluid. The additional of nanoparticles in the base fluid could be stable in the case of suction, which encourage the heat transfer property of the fluid.

1.7 Important Parameters

This section will discuss the important parameters used in this study. They consist of the Prandtl number, $\Pr$, Reynolds number, Re, skin friction coefficient, C_f and local Nusselt number, Nu and local Sherwood number, Sh.

1.7.1 **Prandtl number,** Pr

The relationship between velocity and thickness of the layer temperature is described by the Prandtl number. Dimensionless number is given by

$$\Pr = \frac{\mu c_p}{k} = \frac{\mu\rho}{k/(\rho c_p)} = \frac{\upsilon}{\alpha} = \text{momentum diffusion/ thermal diffusivity}$$

where μ, c_p, k, ρ, υ are the dynamic viscosity, specific heat, thermal diffusivity, fluid density, kinematic viscosity and thermal diffusion coefficient (Anderson et al., 1984). Therefore, the gas with Pr ≡1, the transfer of momentum and energy through the process of diffusion are comparable. For oil, the diffusion momentum is bigger than thermal diffusion, but for liquid metals, Pr << 1, the situation is quite the opposite (Ozisik, 1985). Prandtl number to be measuring the relative size of the velocity boundary layer with thermal boundary layer compared with the numbers of other dimensionless Prandtl numbers can be characterized as one of the fluid properties. Large Prandtl number gives the impression that the fluid is viscous, and vice versa (Sherman, 1990).

1.7.2. **Reynolds number,** Re

Reynolds number is a dimensionless number. This number was introduced by a member of the British fluid mechanics, Obsorne Reynolds, in 1883 . Reynolds number Re is defined as the ratio of inertial to viscous forces given by the following equation

$$\mathrm{Re} = \frac{U_\infty L}{\upsilon} = \text{inertia forces/viscous forces}$$

where U_∞, L and υ are the free flow velocity, characteristic length of the body and the kinematic viscosity.

For most fluids of interest in the studies of fluid mechanics, the Reynolds number is very large, which is between 10^3 to 10^8 (Schetz, 1993). When $\mathrm{Re} \geq 2000$, the fluid flow is strongly influenced by the inertial force said have a place in turbulent flow fields . On the other hand when $\mathrm{Re} < 2000$, the flow is usually laminar fluid (viscous) and it should be eliminated by turbulent viscous friction . In other words, increased fluid viscosity causes the fluid flow to be constant. Changes from laminar flow to turbulent flow depends on the manner and nature of the flow moves.

1.7.2 Local skin friction coefficient, C_f

Local skin friction coefficient is given by

$$C_f = \frac{\tau_w}{\rho u^2(x)/2}$$

where the skin friction can be expressed as

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

The friction component imposed by fluid towards a corpse was parallel with relative motion fluid. The local skin friction displacement is assumed to have experienced thermal transfer along the surface of the body.

1.7.3 Nusselt number, *Nu*

Nusselt number is given by

$$Nu = \frac{qL}{k} = \frac{q\Delta T}{k\Delta T / L}$$

Where q, k, L and ΔT are the heat transfer coefficient, thermal conduction, characteristic length and the reference temperature difference between the surface temperature of the walls of the fluid. Thus, Nusselt number may be interpreted as the ratio of heat transfer by convection to the conduction across the fluid layer thickness L. Based on this interpretation, $Nu=1$ implies the absence of convection, and the heat transfer is performed by pure conduction. Meanwhile large Nusselt number implies an increase in convective heat transfer (Ozisik, 1985).

1.7.4 Sherwood number, *Sh*

The Sherwood number representing

$$Sh = \frac{x h_m}{D_B (C_w - C_\infty)}$$

in which h_m can be expressed as

$$h_m = -D_B \left(\frac{\partial C}{\partial y} \right)_{y=0}$$

where q_w and D_B, are defined as the wall mass flux on the plate and Brownian diffusion coefficient.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Nanofluid is a fluid that contains nanoparticles. Nanoparticles typically in nano sized particles which in diameter sized 1-100 nm. The nanofluid can be in the sphere shape, rod, or individual scattered. The dispersion of solid particles in fluid are mostly been used in thermal investigation because they have good in thermal conductivity and heat transfer rate in the fluid. The nanofluid structure contain thin outer layer called nano layer, inner layer called nanoparticles and the base fluid. The nanoparticles include metals, oxides, carbides, or carbon nanotubes. The common of base fluid examples are water, ethylene glycol and oil. The fluid is carried the nanoparticles in suspension that called colloidal suspension. The nanofluid has high thermal conductivity and heat transfer rate even at very low solid concentrations (<1%). This is because of the dispersion of solid particles in the fluid which cause the thermal conduction of nanoparticles higher (100-1000 bigger from the other fluid). According Choi et al. (2001) small additional of nanoparticles in fluid, can increase the thermal conduction although less than 1% because the particles are extremely small and they can act as fluid molecule.

There are several models which are been investigated for thermal effective conductivity, such as Khanafer et al. (2003) Buongiorno (2006), Tiwari and Das (2007), Nield and Kuznetsov (2009), and Kuznetsov and Nield (2010). Among this model, the

Buongiorno (2006), and Tiwari and Das (2007) are most popular model used by researchers. However this study will focus in Buongiorno (2006) model.

2.2 Buongiorno Model

This model indicated the heat transfer is commonly about the motion of nano solid particles in the base fluid. Thus to test the validity of this assumption, Buongiorno (2006) considered seven slip mechanisms that can produce the velocity of the nano solid particles, such as inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity. However only Brownian diffusion and themoporesis could be consider as slip mechanism in a nanodfluid.

According Buongiorno (2006), Brownian diffusion is a random motion of nanoparticles in the base fluid. The continuous collision between nanoparticles and the fluid contribute Brownian motion.

Buongiorno (2006) stated thermophoresis is known when the particles that can be diffuse under the temperature gradient, where the particles would be move from the hot region to cool region. It is happen when the particles in hot region have more kinetic energy rather than the cool region, thus the particles can be moved.

2.3 Tiwari and Das Model

In Tiwari and Das (2007) model, they considered the behavior of nanofluid due to nanoparticles volume fraction. Tiwari and Das (2007) noted that nanoparticles immersed in