Finite Difference Formulation of Shape Detection Using Poisson's Equation

by

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LIST OF ABBREVIATIONS

PDE	Partial Differential Equation
FDM	Finite Difference Method
2D	Two Dimensional
3D	Three Dimensional
HPM	Homotopy Pertubation Method
BEM	Boundary Element Method
GM	Gernike Moments
GFD	Generic Fourier Descriptor
WZMD	Wavelet Zernike Moment Descriptor
ZM	Zernike Moments
MZM	Modified Zernike Moments
CHT	Circular Hough Transform
GMM	Gaussian Mixture Model
IoU	Intersection of Union
SOR	Successive Over-Relaxation

LIST OF SYMBOLS

ρ	Charge density
ε_0	Permittivity of free space
Δx	Spatial step size at x - axis
Δy	Spatial step size at y - axis
$\nabla^2 u$	Potential 2D Poisson's equation
u_{xx}	Second derivative Poisson's equation at variable x
u_{yy}	Second derivative Poisson's equation at variable y
$u_{i,j}$	Central point
$u_{i,j-1}$	Lower spatial step point
$u_{i,j+1}$	Upper spatial step point
$u_{i+1,j}$	Forward spatial step point
$u_{i-1,j}$	Backward spatial step point
k	Previous iteration value
$k+1$	New iteration value
h	Step size

Formulasi Beza Terhingga untuk Pengesanan Bentuk Menggunakan Persamaan Poisson

ABSTRAK

Objek boleh dikesan atau dibandingkan persamaannya dengan bentuk yang lain, apabila pengesanan bentuk berjaya diterangkan dalam penglihatan komputer. Dalam kajian ini, tiga bentuk geometri telah digunakan dalam mengesanan bentuk iaitu segi empat sama sisi, bulatan dan elips. Persamaan Poisson dalam dua dimensi (2D) digunakan untuk pengesanan bentuk kerana persamaan Poisson boleh digunakan untuk mengesan pelbagai jenis bentuk. Dalam usaha untuk menganggar persamaan Poisson, proses pendiskretan dijalankan dengan menggunakan teknik beza terhingga tersirat. Kaedah Jacobi dan Gauss-Seidel telah digunakan untuk menyelesaikan persamaan linear bagi persamaan Poisson. Algoritma berangka telah dibangunkan dalam perisian MATLAB R2012b. Dalam proses penyelesaian, kaedah Gauss-Seidel menggunakan bilangan lelaran yang kurang berbanding kaedah Jacobi untuk mengesan segi empat sama sisi, bulatan dan elips. Penyelesaian berangka bagi kaedah-kaedah ini telah dibandingkan dan keputusan yang diperolehi adalah berkesan dan sesuai untuk memperoleh bentuk asal. Keputusan simulasi persamaan 2D Poisson telah berjaya meramalkan untuk pengesanan bentuk.

Finite Difference Formulation of Shape Detection Using Poisson's Equation

ABSTRACT

Object can be detected or compared its similarity to other shapes, when shape detection is successfully described in computer vision. In this study, three geometry shapes are used in shape detection which are square, circle and ellipse. The two dimensional (2D) Poisson's equation is used to detect these shapes because Poisson's equation can be used for various types of shape detection. In order to approximate the Poisson's equation, the discretization process is conducted using an implicit finite difference method. The Jacobi and Gauss-Seidel methods have been used to solve linear equation of 2D Poisson's equation. The numerical algorithms are developed in MATLAB R2012b software. In the solution process, the Gauss-Seidel method used less number of iterations compared to Jacobi method to detect square, circle and ellipse. Numerical solutions for these methods are compared and the results obtained using these methods are found to be efficient and suitable to obtain the original shape. The simulation results of 2D Poisson's equation have been successfully predicted for the shape detection.

CHAPTER 1

INTRODUCTION

1.1 Introduction

Object detection, classification and recognition are an importance part of any vision based computer application and has received a great understanding in the recent years. Computer vision is a wide field that include method for capturing, pre-processing, analysing and understanding images become valuable task to detect the object in the image (Sonka, Hlavac and Boyle, 2014). The purpose of object detection in this study is to solve an object with their geometrical shape. Bai, Yang, Yu and Latecki, (2008) have said in detection and recognition of contour parts based on shape similarity, object can be classified using colour, texture, shape, movement and location. Shotton, Blake and Cipolla (2008) have investigated on multiscale categorical object recognition using contour fragments, and Felzenszwalb and Huttenlocher (2000) has investigated on efficient matching of pictorial structure said that, shape of object commonly can be described from the shape of boundary edges or contour, while extracting sufficient shape information still poses a large challenges recent research in detection allows extraction of local shape hypothesis. Xie, McGinnity and Wu (2010) have studied on automatic extraction of shape features. It has been revealed that a lot of approaches used various geometry classification to present the shape such as circularity, concavity, convexity, principal axis ratio and so on. The shape detection for square, circle and ellipse can be done by using Poisson's equation as have been discussed by Gorelick, Galun, Sharon, Basri and Brandt (2006).

Gorelick et al. (2006) have investigated the useful of Poisson's equation to classify and recognize two-dimensional (2D) object. The author has said that, Poisson's equation has several useful properties that can be used for detection and classification task. It can be used to extract a wide variety of useful properties of silhouette binary image including segmentation of silhouette into parts, identifying corners of various resolution scales, deriving a skeleton structure and locally judging the orientation. In this study, Poisson's equation is used for solving shape detection process. Poisson's equation has several properties that can be used to extract a vast variety of useful properties of a geometrical shape. Besides Gorelick et al. (2006), Haidar, Bouix, Levitt, McCarley, Shenton and Soul et al. (2006) also believed that Poisson's equation can be used for shape measurements. Such a solution allows the smooth nature and uniqueness robust computations, and the discrete finite number of local maxima suggests its use for divided shape into parts. The function of this representation in two dimensional is illustrated with geometry shape of natural objects. The 2D Poisson's equation is used to solve shape detection problems with special numerical method.

Our detection process is based on shape properties of objects, in which the shape is identified using their geometrical shape pattern and implemented using MATLAB R2012b. MATLAB R2012b is really efficient and has fast processing and less time consumption for performing computer vision task. One of the most commonly used in numerical methods for solving partial differential equations is finite difference method, which is still popular used at nowadays in research study (Liu, Gao and Zhang, 2010). The proposed method has the better detection performance for detect geometrical shape. In generally, the discretization of finite difference brings to large sparse linear systems which are commonly resolved by iterative method. Jacobi and Gauss-Seidel methods are iterative solvers that have excellent numerical properties (Young, 1972). Jacobi and Gauss-Seidel methods are used in this study because both methods are much more efficient for solving linear equation of Poisson's equation for shape detection problem. The chosen of the methods also has related with what Bruaset, Cai, Langtangen and Tveito (1997) and Sintay (2008) have said that, the best way to solve Poisson's equation is by using a finite difference method (FDM).

1.2 Problem Statement

Every minute of lives, people used shape detection to detect objects. Normally, objects can be detected easily by people, but computer vision system cannot perform the shape detection as good as the human. The system needs a lot of information to simulate the detection. There are a lot of information that can be used to make the system efficiently recognize the objects and detect by colour, texture, shape and others. Poisson's equation can be used to extract various properties of a shape including its part structure and rough skeleton, local orientation and aspect ratio of different parts, and convex and concave sections of the boundaries. Specific finite difference method is used to solve the Poisson's equation for shape detection.

1.3 Objectives

The main objective of this study is to predict the shape of detection shape. In order to achieve the main objective, the following sub-objectives will be carried out:

1. To discretize 2D Poisson's equation using FDM.
2. To develop numerical algorithm and procedure for solving 2D Poisson's equation.
3. To test and compare efficiency of numerical solution with 2D test dataset that includes geometric shapes.

1.4 Scope

In this study, the 2D Poisson's equations will be used in solving the shape detection for square, circle and ellipse. Poisson's equation will be approximated by finite difference method.

1.5 Significant of the Study

The significant of this study is the results of the 2D shape detection which is done by numerical modeling based on computer vision that can be used as prediction results for original geometric shape. This will help researchers to identify the problems that may arise before the actual experiment is carried out. Cost, time and energy resources can be optimized. Besides that, the numerical results obtained can be used for comparison result in efficiencies of different iteration methods. If the comparative results are excellent and satisfied, the mathematical model can be used for 3D versions problems. The results of this study can be used as a reference and comparison to researchers in the future.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In this chapter, the existing research on Poisson's equation in solving the shape detection problem is reviewed. After that, the Poisson's equation are explained and discussed. Finally, the discussions about the numerical methods for solving the Poisson's equation are summarized.

2.2 Shape Detection

Shape detection plays an essential part in the study of images. The objects can be detected or compared its similarity to other shapes when shape detection is successfully described. Many researchers used different ways to detect objects in their works. Among these researchers are Biederman (1985), Marr and Nishihara (1978), and Sebastian, Klein, and Kimia (2002) who proposed the ways to detect object by computer vision systems. In computer vision, shape detection is done by simulation from a given image. Human can detect multiple of objects in images with different form of color, texture, and others. The advantage in the development of this field has been reported by Sonka et al. (2014) in image processing, analysis and machine vision is to duplicate the abilities of human vision by electronically perceiving and understanding. The detection of any type of objects in image processing needs to differentiate the desired object from other objects in the scene or in the background. The shape information that existing in the image in the form of coherent image regions or segments and their bounding contours, provide a powerful clue that can be used for detection. Generally, until nowadays researchers still trying to solve shape detection based on computer vision systems.

A detection process contains a feature extraction mechanism that includes numeric or symbolic information from observations process and depending on the extracted features can be used to detect the object. Features can be extracted either from gray scale image or from color image. Gode and Khobragade (2016) have been used shape feature to detect the object using proper segmentation algorithm in object detection using color clue and shape feature. The authors of this paper used color information of object shown in Figure 2.1 to segment the object and compared extracted object contour with detected object contour. Anvaripour and Ebrahimnezhad (2013) proposed a novel object detection approach based on local shape information for accurate object detection using local shape descriptors. The boundary fragments and local orientation of objects are utilized as local features shown in Figure 2.2(a) and Figure 2.2(b). Object boundaries are the best features for describing object shape that are invariant to color and brightness changes in images.

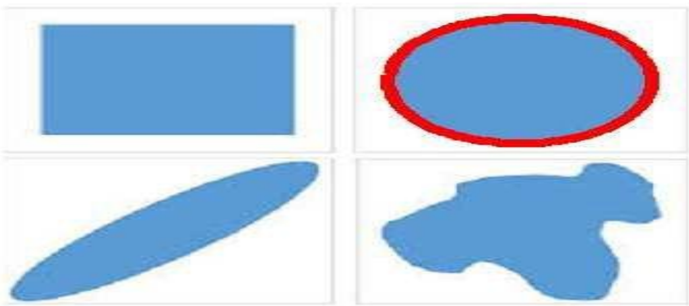

Figure 2.1: Detected image using color information. (Gode and Khobragade (2016))

Figure 2.2(a): Edge fragments of a swan image, with four boundary fragments of swan, Figure 2.2(b): Four sample fragments extracted from image. Blue, green, red and magenta fragments have orientations [90, 90], [0, -30], [0, 30] and [-30, -60]. (Anvaripour and Ebrahimnezhad (2013))

Besides that, detection process also can be used for counting the number of the objects in the image automatically. Ni, Khan, Wang, Wang and Haider (2016) have been proposed this approach on automatic detection and counting of circular shaped overlapped objects using circular hough transform and contour detection. The authors have been tested an image shown in Figure 2.3 that has some circular shaped overlapping objects while one circular and non-circular shaped objects are overlapped with each other. The work is implemented using opencv3.0 libraries linked with visual studio 2012. Arteta, Lempitsky, Noble and Zisserman (2013) have studied on learning to detect partially overlapping instances said that the overlapping object in the scene makes the task challenging to be completed. The distinguishing of the desired object from other objects in the scene or in the background is necessary for the detection process.

Figure 2.3: The image used in the experiment. (Ni, Khan, Wang, Wang and Haider (2016))

Besides that, feature extracted also can be used on classification problem. Xie et al. 2010 have proposed automatic extraction of shape feature for classification of leukocytes. The number of segments of nucleus and the shape of segment of nucleus are important features for classification process. The type of leukocytes shape is shown in Figure 2.4. Tycko, Anbalagan, Liu and Ornstein (1976) and Pavlova, Cyrrilov and Moumdjiev (1996) have studied on leukocytes and also revealed that the colors of nucleus and leukocyte cytoplasm are the ways to clasify leukocytes. A limited accuracy is provided only for used of these ways. On the other hand, Mircic and Jorgovanovic (2006), Sabino, Costa, Rizzatti and Zago (2004), and Ramoser, Laurain, Bischof and Ecker (2006) used the size, shape and segmentation of nucleus to detect the existence granules in cytoplasm and structured them to increase the performances among the classifiers.

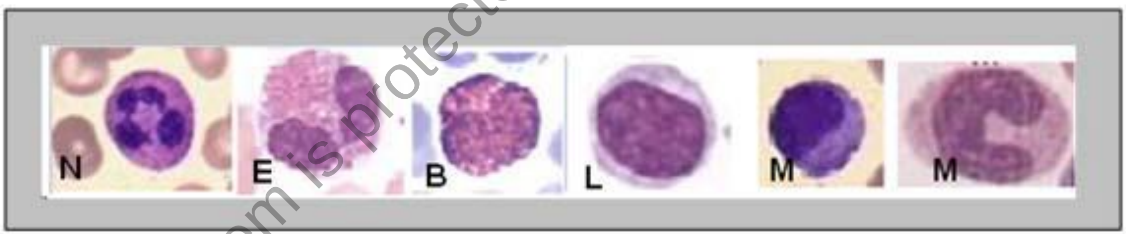

Figure 2.4: Examples of 5 types of human leukocytes. (Xie, McGinnity and Wu (2010))

Shape classification also done in oncology field Hardisty, Gordon, Agarwal, Skrinskas and Whyne (2007), Guyon, Foskey, Kim, Firat, Davis, Haneke, and Aylward, et al (2003), and Wang, Eirini Karamani, Erickson, Uday and Vasileios (2007) who studied in the field of oncology revealed that shape of the brain tumor is an important information that is difficult to be measured. The right classification of the tumor shape has the potential to provide clinicians with data such as disease type, stage, progression or regression and outcome. Shape becomes even more essential in the radiation oncology especially on identified, quantified, and tracked. Thus, there is a requirement to measure tumor shape in order to provide a significant amount of accuracy and stability. The brain tumor shape is shown in Figure 2.5.

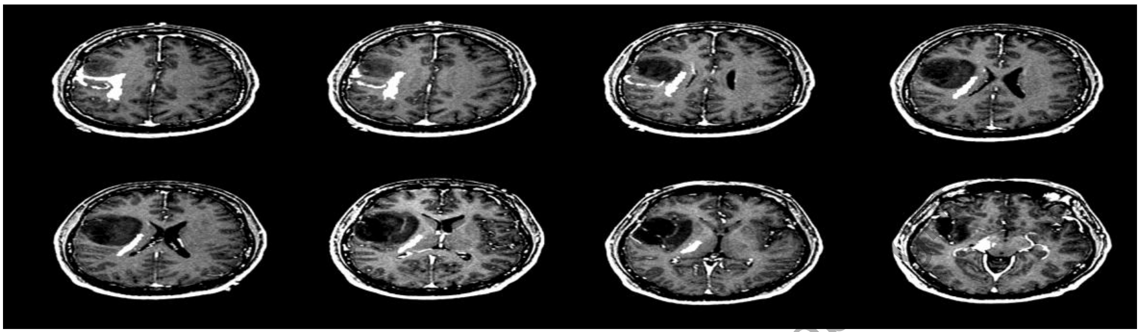

Figure 2.5: The examples of the brain tumor. (Cha, S. (2006). Update on brain tumor imaging: from anatomy to physiology. *American Journal of Neuroradiology*, *27*(3), 475-487.)

In this study, some 2D geometry shapes are detected by using 2D Poisson's equation. There are various types of geometry shapes such as square, ellipse, circle, triangle, and pentagon and so on. Based on related work by Haidar et al. (2006), Poisson's equation can be used for various types of shape detection. This is because Poisson's equation is implemented to solve the problem in this study. The 2D geometry shape is shown in Figure 2.6.

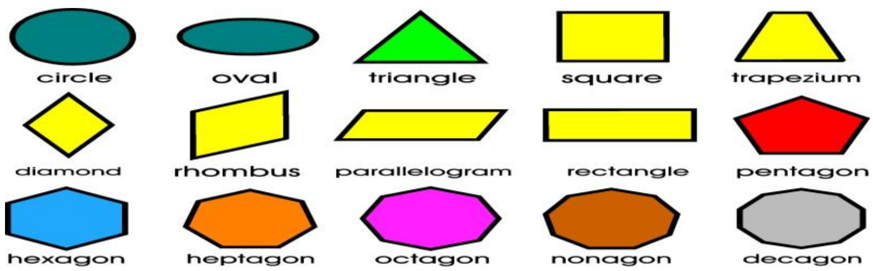

Figure 2.6: The examples of 2D geometry shapes. (Nor (2015) 6th class science (Geometry in Primary mathematics))

2.3 Poisson's Equation

Tveito, Bruaset and Lysne (2009) have noted that, Siméon Denis Poisson (June 21, 1781-April 25, 1840) published Poisson equation in 1813 in the *Bulletin de la société philomatique* as an improvement of an equation published by Pierre-Simon Laplace in 1827. Poisson's equation is a second order elliptic partial differential equation (PDE) with full performance in electrostatics, mechanical engineering, and theoretical physics. Electrostatics is the part of physics which considered with the forces utilized by a static electric field consequent to charge objects. In the field of electrostatic, Poisson's equation is directly derived from the first Maxwell Equation by Carl Friedrich Gauss in 1813 for electric field, u as:

$$\nabla^2 u = -\frac{\rho}{\varepsilon_0} \tag{2.1}$$

where ρ is the charge density and ε_0 is the permittivity of free space. If $\rho = 0$, Equation (2.1) becomes a Laplace equation. For two dimensional spaces is, the Poisson's equation becomes

$$\nabla^2 u = u(x, y) = u_{xx} + u_{yy} = -\frac{\rho}{\varepsilon_0} \;, \tag{2.2}$$

where x and y are coordinates within the interior of an object.

The Equation (2.2) can be used to represent more than just electrostatics function and then using the form of potential Poisson's equation, u as

$$\nabla^2 u(x, y) = u_{xx} + u_{yy} = -1 \tag{2.3}$$

In this study, Equation (2.3) is used and the discretization processes for 2D Poisson's equation will be carried out in Chapter 3.

Poisson's equation has been presented by Gorelick et al. (2006) in shape representation and classification. The authors pre-segmented the shape of silhouettes to characterize silhouettes using Poisson's equation. Figure 2.7 shows the result of using Poisson's equation for producing silhouettes shape. Solution of Poisson's equation at a point in the interior of a region represents the average time required for a particle to hit the boundary at a random walk starting at a specific point. Such a solution allows the smooth propagation of contour information of a region to every internal pixel. In shape based detection and top–down delineation using image segments investigated by Gorelick and Basri (2009) said that Poisson's equation is solved for a big region with a tall effective boundary to describe shape of the region and to find proper segment of the desired object.

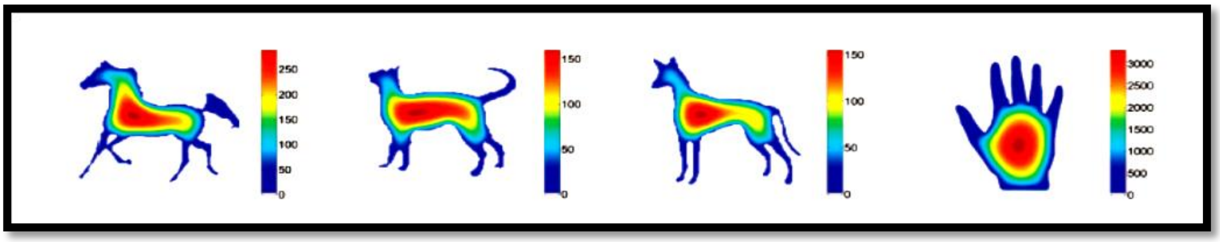

Figure 2.7: Result of using Poisson's equation for producing silhouettes shape.
(Gorelick, Galun, Sharon, Basri and Brandt (2006))