Monitoring Product Quality In Cliching Process: Comparison Between Univariate and Multivariate Control Charts

By

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CHAPTER 1

INTRODUCTION

1.1 Introduction

This chapter describes the general matter for this research. It consist of the background of the problem, statement of the problem, objectives of the study, the scope of the study.

1.2 Background of Research

Clinching is a method of joining different metal parts (mainly sheets) by a process of local deformation without the use of any additional joining elements with the application of a punch and a die. The mechanical interlock with application the clinching technique is shown schematically in Figure 1.1. During clinching, a punch presses the joined sheets inside a die cavity forming a shape that locks the sheets together (Sadowski & Balawender, 2011).

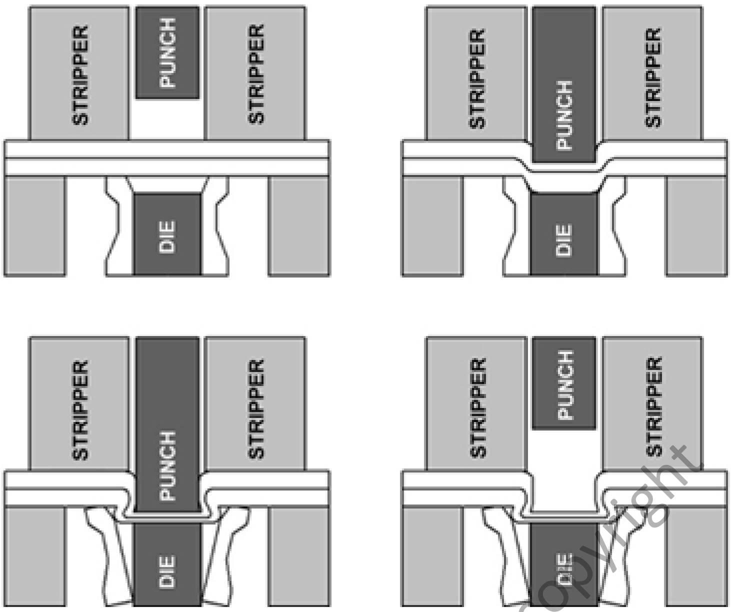

Figure 1.1: Clinching technique.

The issue of quality among the small to medium-sized (SMIs) automotive parts manufacturing and supply companies in Malaysia has been discussed for the previous years. It is no longer a compromise to suppliers or else it will cause as a major risk to the national automotive industry. There have been pressures from the local SMI suppliers to improve the overall quality of parts supplied to the Malaysian pioneer car manufacturer (Wan Mohamed, Cha, & Chin, 2006; Mohd Najib, 2006; Mohd Nor, 2006). From previous studies on the Malaysian automotive industry have shown that, the statistical process control tools have been used in the local automotive parts suppliers to improve their quality (Jafri, Sha'ri, & Ismail, 2007; Noviyarsi, & Sha'ri , 2004; Salimah, 2001).). However, many of these SMIs automotive parts suppliers are still facing the same problem according the longstanding issue of low quality products (Leete, 2007).

In the international arena, studies on the application of control charts in the stamping of automotive panels have been evident for the last two decades, gradually progress from univariate to multivariate applications (Wang, 1995; Yang, 1996; Yang, & Trewn, 2004). Other studies on automotive part stamping demonstrate the use of principal component analysis and some extend to another multivariate application of experimental design. (Ceglarek 2000; Kuzrna-smith, 2000). The example of the previous case study about stamping process. This study embarks on a statistical process control by specifically proposing a multivariate control chart scheme in monitoring the quality of stamping process of an automotive panel based on its geometrical dimensions. The need for a multivariate approach of control charting owes mostly to the multivariate nature of quality variable characterizing the manufacturing process as well as product dimension. Quality variables, namely, surface, trim, and hole eccentricity characterize the dimensional measurement of the panel. Even in a narrow sense, a single quality variable itself, for instance, the surface is narrated by several different measure points. Monitoring large number of univariate control charts are almost impossible. Apart from the multiple nature of quality variables, these variables periodically demonstrate some form of correlation, either cross correlation or serial correlation, or even both (Hammett, P., Baron, & Smith., 2000; Yang, & Trewn, 2004). Managing separate univariate control charts, again, would not be able to take into account the correlation factor. A multivariate approach would, then, best serve the purpose of quality monitoring of automotive stamping processes.

From this issued, I have made a decision to investigate the clinching process. The comparison result will obtain between univariate and multivariate to see the sensitivity of the shift. In this study, multivariate control chart is one of the proper approaches to monitor the clinching process quality in automotive industries. The Hotelling's T^2 control chart technique is most appropriate when a multivariate control chart technique is applied for the first time to monitor the quality variables. This study was focused on using a control chart, especially Hotelling's T^2 to monitor the process target and variability.

1.3 Problem Statement

Clinching is a simple technique for point joining of metal sheets from around 0.5 to 3mm thick, up to a total joint thickness of about 6mm. Clinching is used mainly for high-volume, undemanding applications automotive components. The example of automotive components that I have study is High Brightness LEDs Lamp.

In the factory that I visited, they are using a univariate control chart to monitor the quality characteristics, for example cap thickness 1, cap thickness 2 and pull test. Basically, univariate deals with 1 predictor variable, but multivariate deals with multiple predictor variables. In this case, there has a correlated issue between quality characteristics. The potentials of multivariate approach to quality monitoring is it can monitor the quality characteristics simultaneously while univariate only can monitor single variable. In this dissertation, the multivariate control charts were used to monitor the quality characteristics. The comparisons were obtained between the univariate and multivariate methods.

1.4 Research Objectives

The aims to do the research are:

(a) To investigate the limitations of univariate tools for process target and variability monitoring.

(b) To study multivariate tools for monitoring process target and variability.

(c) To determine the importance of multivariate control chart in the automotive clinching industry.

1.5 Research Scope

There are two scopes:

1. Theoretically:

   a) There are three scenarios of multivariate process under considerations.

      i. The observations are independent and identically distributed (i. i.d).

      ii. The quality characteristics are correlated.

      iii. The observation are distributed as multivariate normal with mean vector μ and covariance matrix $\sum$.

2. Applications

   To illustrate the advantage of the multivariate control chart in automotive clinching industrial process.

1.6 Significant of the Study

Now, the manufacturing process quality becomes more and more complex. Control charts can actually allow companies to determine when something unexpected occurs in their manufacfuing process. Without having this kind of information in hand, not only problems are discovered when it is already too late, but it is more difficult to determine the cause. This study specifically demonstrates the application of multivariate statistical technique through its diagnostic features such as combining measurements from many different characteristics and simultaneously.

The univariate only can be used on one variable and have limitations for many variables. In this case, there has a correlated issue between quality characteristics. The potentials of multivariate approach to quality monitoring is it can monitor the quality characteristics simultaneously while univariate only can monitor single variable.

This is the reason why multivariate has been used to control the process quality in manufacturing. Monitoring process target and variability are two important parameters to control the quality by using multivariate methods. The stability of the product depends on the process target and variability.

1.7 Organisation of Dissertation

In this dissertation, Chapter 1 covers the general material regarding this research. Chapter 2 describes the theories that are useful for this research. In Chapter 3, the steps by steps of data collection and compiling the data set are performed. The result obtains in this chapter is needed in the next chapter. Chapter 4 is the most important part of this dissertation to discuss about the covariance shift by using a multivariate control chart. It shows that the advantage of multivariate control charts in the automotive clinching industry. The last chapter is Chapter 5, which contains a conclusion of the dissertation and the recommendations for further research.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter present a review an univariate and multivariate control charts and their applications in the automotive industries.

2.2 Statistical Process Control

Statistical Process Control (SPC) is the application of statistical methods to monitor and control a process to operate at full potential in order to produce a conforming product. The benefits of SPC include improving productivity, reduced scrap, rework, reduce costs and obtain higher customer satisfaction. The SPC techniques, when applied to measurement data, can be used to highlight production areas that would benefit from further investigation to improve the quality. The techniques enable the user to identify variation within the process. Understanding process variation is the first step towards quality improvement.

A process is considered stable if it meets customer demand, while it is considered unstable if it does not meet customer demand. To get the stability of a process, the process must be able to operate with a little variation from original target. SPC is one of the effective ways to control and minimize the variation to obtain a stable process. SPC can also be used to improve the ability of the products produced by reducing variability (Montgomery, 2009).

Many processes can use the SPC. The effects of the use of SPC will create an environment in which all individuals in an organization will indirectly involve in the process of improving the quality and productivity continuously. This will be easier when the management is involved to create this environment. By creating this environment, it will be easier for an organization in a business to achieve its quality improvement the desired goals. Online statistical process control procedures using control charts are much easier to control and achieve the goals set (Montgomery, 2006).

2.3 Control Charts

Control charts is a tool for monitoring production process in order to focus on continuous improvement. Any unsual variation if exist will be further investigated and thus process modification will be done . As a result, control chart provide us the opportunity of production process improvement in order to reduce defects. Shewhart control chart is the common control chart that was used for monitoring quality characteristics. Control chart is a graphical representation to identify the condition of a process whether it is in-control or out-of control (Montgomery, 2009).

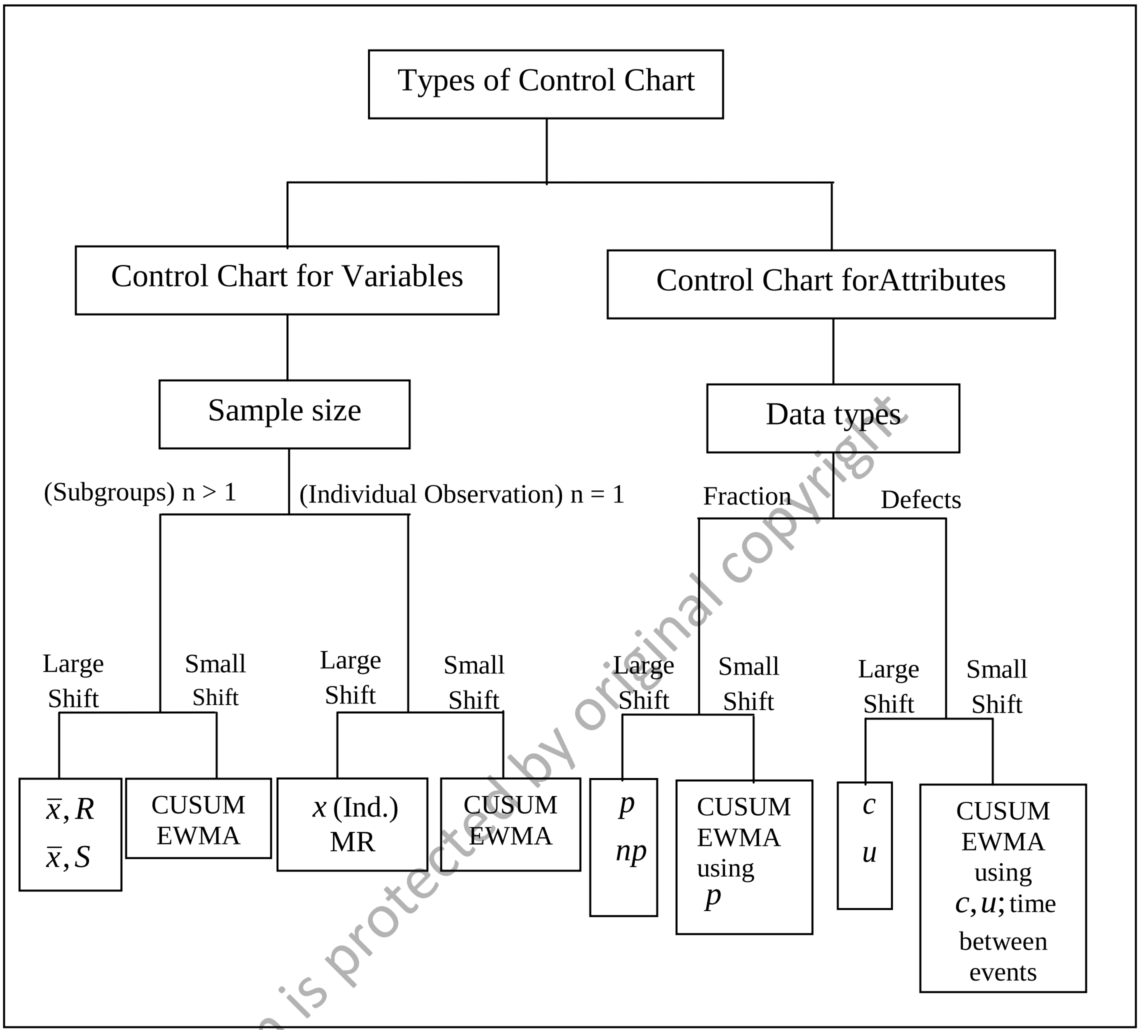

Figure 2.1: Control charts for qualitative and quantitative quality characteristics..

This study deal with variables control charts. They are too important parameters to be monitored in variables control charts. The parameters are referred to location and spread parameters. Location chart is used to monitor the process mean while dispersion chart is used to monitor the process spread or process variability. Both control charts are used for detecting any changers in the process parameters or also call as quality characteristics. The process changers may occur not only in the process mean but as well as in the process variability. Thefore, it is essential to monitor and control both of them using control chart. In this study, we construct univariate control charts and multivariate control charts for independent and identically distributed rational subgroup in order to monitor the process mean and process variability. The development of control charts is due to the fact that variations always exists. Thus, it is important to monitor the process behaviour so that the process is stable and predictable. Only then quality of product can be achieved. Evaluating process stability require us to identify and eliminate sources of variation that may effect our process. There are two sources of variations; common cause and special cause.

2.4 Common Cause and Special Cause Variation

In manufacturing process, there are two types of variations: common cause and special cause. Common cause variation is the natural inherent variation in the process. Examples of common cause variation are wrong procedures, lack of a clear standard operating procedures, the working environment is not suitable, the machine is not suitable for the job, the raw material is not suitable, the error of measurement of vibration in industrial processes, ambient temperature and humidity, lack of training, normal wear and tear and diversity in the vicinity. The process is considered stable if it is independent and identically distributed if only common cause present (Montgomery, 2009).

Special cause is a cause of variation which is not an inherent part of a process, but arises out of intermittent, unpredictable and unstable factors. These extraordinary causes are indicated by data points that fall outside of the limits of a control chart. For examples in special causes are not present operators, poor tuning equipment, the operator is asleep, the operator does not work, mechanical errors, computer crashes, and vulnerable groups of raw materials and power surges. The process out of control when special causes of variation are present (Montgomery, 2009).

Thus, SPC is very useful in quantifying variation in a process. SPC tools such as control charts help us to detect and separate both common cause and special cause variation.

2.5 Univariate Control Charts for Monitoring Process Mean : $\bar{x}$ Chart

Univariate control charts are based on the measurements of only one individual characteristic. The univariate control charts are widely used in manufacturing industries and make up an important part of quality improvements programs (Montgomery, & Wadsworth, 1972). Walther S. Shewhart first proposed the general theory of statistical control charts in the early 30's of the last century. A typical control chart is a graphical display of a quality characteristic that has been measured from a sample versus the sample number or time (Figure 2.2).

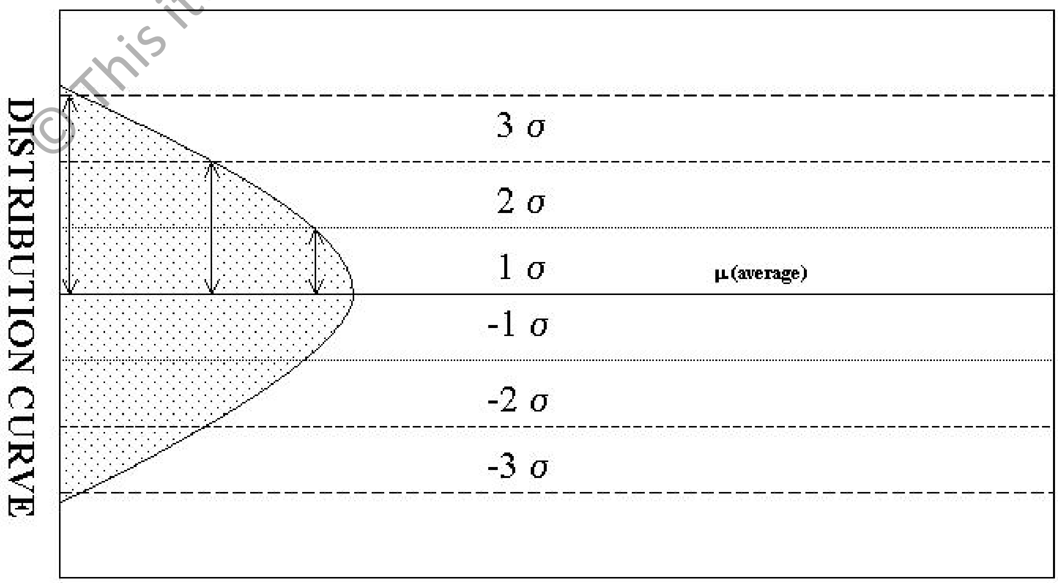

Figure 2.2: Example of a Shewhart $\bar{x}$ control chart.

In Figure 2.2, three lines can be distinguished. The centerline represents the average value (μ) of the quality characteristics. This value corresponds to the in-control state. The other two horizontal lines are the upper control limits (UCL= $\mu + 3\sigma$) and lower control limits (LCL= $\mu - 3\sigma$). As long as the points fall between the UCL and LCL, the process is said to be statistically in-control. The opposite is true if a point plots outside these control limits. The process is said to be out of control. Further action is required to locate the cause of this abnormal deviation from the in-control situation.

It is important to note that it is assumed that $\bar{x}$ is normally distributed due to the central limit theorem. Also, it is assumed that the process mean and standard deviation are known. Closely related to the $\bar{x}$ chart is the $\bar{R}$ chart (monitor the sample range) and S-chart (monitoring the sample standard deviation). The disadvantage of the $\bar{x}$ chart is that small changes of the mean are not detected. Only shifts in the mean of magnitude 1.5σ to 2σ or larger are effectively detected.

Suppose that m sample of each containing *n* observations on *p* quality characteristics are available. Let the sample mean of the i^{th} sample be $\bar{x}_i$ for i=1,2,3..,m. Then we estimate the mean of the population μ as follow:

$$\mu = \bar{\bar{x}} = \frac{1}{m}\sum_{i=1}^{m}\bar{x}_i \quad (2.1)$$

The formulas for constructing the control limits on the $\bar{x}$ chart are:

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R} \quad (2.2)$$

$$\text{CL} = \bar{\bar{x}} \quad (2.3)$$

$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R} \quad (2.4)$$

The constant A_2 is presented for several sample sizes in Appendix Table VI.

2.6 Univariate Control Charts for Monitoring Process Variability : R Chart

Using control charts in the range, the standard deviation of each subgroup can be computed through a more modern approach to the monitoring process diversity using the values of the standard deviation (σ). This is called *R* chart. Using the standard deviation of the R chart, control limits of control charts can be developed and it is normal. Normally, the sample size used for subgroups is small (less than 10). Nevertheless, R chart is used quite commonly as commonly used in computer software (Montgomery & Runger, 2003).

Let $R_1, R_2, \ldots, R_m$ are the sample rangers of the *m* samples. The average range is:

$$\overline{R} = \frac{1}{m}\sum_{i=1}^{m} R_i \quad (2.5)$$

The center line and upper and lower control limits of the *R* control chart are:

$$\text{UCL} = D_4\overline{R} \quad (2.6)$$

$$\text{CL} = \overline{R} \quad (2.7)$$

$$\text{LCL} = D_3\overline{R} \quad (2.8)$$

The constant D_3 and D_4 are presented for several values of *n* in Appendix Table VI.

2.5.2 CUSUM Chart

The cumulative sum, or CUSUM chart, is a good alternative to detect small shifts of the process target. Consider a control chart for the mean with a target for the process mean μ_0, with the following observations $x_1, x_2, x_3, ..., x_i$. The CUSUM are calculated according to:

$$\left.\begin{aligned} S_1 &= x_1 - \mu_0 \\ S_2 &= S_1 - (x_2 - \mu_0) \\ S_3 &= S_2 - (x_3 - \mu_0) \\ S_i &= S_{i-1} - (x_i - \mu_0) \end{aligned}\right\} \qquad (2.9)$$

Or

$$S_i = \sum_{a=1}^{i} (x_a - \mu_{0)}) \qquad (2.10)$$

Often, the starting value for the CUSUM chart is set to zero. Small changes in the mean will result in a relatively large increase in the slope (positive or negative) of the CUSUM chart.

2.5.3 EWMA Chart

Univariate Shewhart chart has a weakness for just taking the last data point and does not bring the memory of the previous data. This leads to small changes in the mean of a random variable can not be detected quickly. Exponentially weighted moving average (EWMA) charts flawed one way to make the detection of small process shift. EWMA has many advantages that make it attractive as it can quickly detect small changes in the quality characteristics. Roberts (1959) has introduced EWMA charts to make quick detection of small changes in the mean. It is used in time series forecasting model broadly to the process by pulling gently (Box, Jenkins, & Reinsel, 1994). EWMA can provide predictions for the future. So, it will be able to provide a mechanism for the control of dynamic processes (Hunter, 1986).

The Exponentially Weighted Moving Average (EWMA) is a time-weighted control chart that plots the exponentially weighted moving average. The EWMA charts are specially suited to monitor processes that exhibit a drifting mean over time, or for detecting small shifts in a process. For the Shewhart chart control technique, the decision regarding the state of control of the process at any time, *t*, depends solely on the most recent measurement from the process and, of course, the degree of "trueness" of the estimates of the control limits from historical data. For the EWMA control technique, the decision depends on the EWMA statistic, which is an exponentially weighted average of all prior data, including the most recent measurement (Box, Jenkins, & Reinsel, 1994).

By the choice of weighting factor, λ, the EWMA control procedure can be made sensitive to a small or gradual drift in the process, whereas the Shewhart control procedure can only react when the last data point is outside a control limit.

According to Robert (1959), the EWMA statistic given by:

$$EWMA_t = \lambda Y_t + (1-\lambda) EWMA_{t-1} \tag{2.11}$$

for $t = 1,2,\ldots,n$.

where

- $EWMA_0$ is the mean of historical data (target)
- Y_t is the observation at time t
- n is the number of observations to be monitored including $EWMA_0$.
- $0 < \lambda \le 1$ is a constant that determines the depth of memory of the EWMA.

The univariate ($\bar{x}$ chart, CUSUM chart and EWMA chart) are only demonstrate the relationship between independent variable but miss relationship among the dependent variable. From this situation, we want to improve this study from univariate to multivariate because multivariate consider more than one dependent variable at a time.

2.6 Multivariate Statistical Process Control

Multivariate charts typically monitor the mean vector or the variance-covariance matrix. Examples of such multivariate chart Hotelling T^2, Multivariate Exponentially Weighted Moving Average (MEWMA) and Cumulative Sum (CUSUM). Multivariate quality control charts are a type of variables control that how correlated, or dependent, variables jointly affect a process or outcome. The multivariate quality control charts are powerful and simple visual tools for determining whether the multivariate process is in-control or out-of-control. In clinching process, there are several variables need to monitor and the variables are correlated with each other. This is because the process of clinching almost the same with stamping process where to joint the sheet metal elements while the difference is stamping process produce heat effect on the material structure. From previous research, when the Hotelling's T^2 control chart scheme is applied to a new set of sample observations of a selected quality variable of an automotive stamped part, the 'out-of-control' condition is revealed. This finding confirms the need for quality enhancement in the automotive stamped parts manufacturing (Jafri, Sha'ri, & Ismail, 2007). From this finding , we have a proof to apply this method to the clinching process based on the characteristics of both process. The multivariate control chart has several advantages in comparison with multiply univariate charts (Djauhari, 2005):

1. The actual control region of the related variables is represented.
2. A signal control limit determines whether the process is in control.
3. Multivariate control chart simultaneously monitors two or more correlated variables. To monitor more than one variable using univariate charts, we need to create a univariate charts for each variable.
4. The scale of multivariate control charts unrelated to the scale of any of the variables.
5. Out-of-control signals in multivariate charts do not reveal which variable or combination of variables cause the signal.

For this study, cap thickness 1, cap thickness 2 and pull test will be simultaneous monitoring to see the comparison result between univariate and multivariate.

2.6.1 Shewhart Multivariate Control Charts

To build a multivariate Shewhart chart, the first step is to analyze from an initial set of data that is considered to be in control. It is called the Phase I and Phase II. This analysis is important to get the difference between the two phases. In the Phase I, the necessary data is very large for data collection in order parameters and control limits can be estimated for the phase II (Mason, Tracy, & Young, 1997).

2.6.1.1 Hotelling's T^2 Charts

The Hotelling's T^2 control chart is employed to monitor the mean of geometrical dimension of a selected automotive stamped part. The main reasons selecting for the Hotelling's T^2 control charting techniques are due to, firstly, unknown population parameters of the automotive stamped data (Alt & Smith, 1988; Cheng, Away, & Hassan, 2004; Lowry & Montgomery, 1995) and secondly, Hotelling's T^2 control chart is most suitable for the cases where mean shifts in the mean vector are not small (Cheng et al., 2004; Woodall, 2000). Previous empirical studies have shown that automotive stamping process bound to produce large mean shifts in the quality variables of its stamped parts (Hammett et al., 2000). From research carried out on stamping process, it certainly can practice in the clinching process because both of a process involve the study of the quality characteristics.

The multivariate T^2 control charting technique is the extension of the Shewhart univariate procedures. In the univriate statistical control, the normal distribution is used to describe the behavior of a continuous quality characteristic X. The exponent of the probability density function can be written as

$$(x-\mu)(\sigma^2)(x-\mu) \tag{2.12}$$

In a multivariate case where there are p quality variables, $X_1, X_2, ..., X_p$, the p-component vector $X' = \lfloor X_1, X_2, ..., X_p \rfloor$ and the mean vector is given as $\mu' = \lfloor \mu_1, \mu_2, ..., \mu_p \rfloor$. The variances and covariances of a random vector X are presented in the form of a $p \times p$ covariance matrix Σ where the main diagonal and off-diagonal elements are the variances and covariances, respectively. Assuming the distribution of p quality variables are p-variate normal, $N_p(\mu, \Sigma)$, the generalized form of the squared distance from X to μ s the Hotelling's T^2 statistics given as:

$$T^2 = (X-\mu)' \Sigma^{-1} (X-\mu) \tag{2.13}$$

In a retrospective operation, the Hotelling's T^2 statistics can be used to identify outliers in the historical data set, to identify the mean shifts in the new groups of data and other distributional deviations from in-control distributions (Mason, Chou, & Young, 2009).

For the i^{th} sample vector contains onservations on each of the p variables $X_{i1}, X_{i2},..., X_{ip}$, hence, the sample mena vctor is defined as,

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad (2.14)$$

And the sample covariance matrix is

$$S = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)\left(X_i - \overline{X}\right)' \qquad (2.15)$$

The Hotelling T^2 statistics measure the significant shifts from the out-of-control mean vector, μ_s to the nominal mean vector μ_0, as such the larger the T^2 value, the more distant is the observations from the mean vector. This concept is in line with testing the null hypothesis $H_0 = \mu = \mu_0$ vs $H_1 = \mu \neq \mu_0$, the null hypothesis is to be rejected if

$$\left(X_i - \mu\right)' \Sigma^{-1}\left(X_i - \mu\right) > T^2_{p,\alpha} \qquad (2.16)$$

In equation (2.16) above, the covariance matrix is assumed to remain constant as shift in the mean vector are monitored. If μ and Σ are both unknown, they are estimated by $\overline{X}$ as in equation (2.14) and S as in equation (2.15) hence the T^2 statistics is given as (Tracy, Young, & Mason, 1992);

$$T^2 = \left(\overline{X}_i - \overline{X}\right)' S^{-1}\left(\overline{X}_i - \overline{X}\right) \qquad (2.17)$$

The control limits for monitoring future production as below:

$$\left.\begin{aligned} \text{UCL} &= \frac{p(m+1)(n-1)}{mn - m - p + 1} F_{\alpha, p, mn-m-p+1} \\ \text{LCL} &= 0 \end{aligned}\right\} \qquad (2.18)$$

Where p represents the number of quality characteristics, n is the sample size for each group, m represents the number of sample groups and the F-statistic comes from the distribution with the number of degrees of freedom as specified.

Hotelling's T^2 was used for multivariate process monitoring and control. It has a characteristic dependence on the correlation between process variables. If there is no correlation, there is no need to construct Shewhart charts for monitoring the T^2 statistic. If there is a correlation, T^2 can be decomposed into orthogonal components. By using this method, it can show how each variable is related to the remaining variables (Mason et. al., 1997).

2.6.2 Generalized Variance (GV) Chart

Generalized variance is determinate of the sample variance-covariance matrix $|S|$. Jackson (1985) proposed that the starting point of the statistical application of the method of principal components is the sample covariance matrix S for a p-variate problem,

$$S = \begin{bmatrix} S_1^2 & S_{12} & \cdots & S_{1p} \\ S_{21} & S_2^2 & \cdots & S_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ S_{p1} & S_{p2} & \cdots & S_p^2 \end{bmatrix} \tag{2.19}$$

Where*s* i^2 is the variance of the i^{th} variable and S_{ij} is the covariance between the i^{th} and j^{th} variables.

Multivariate process monitoring needs concern on two levels. Monitoring process means vector μ and monitoring the process of variability. Proses variability is summarized by the $p \times p$ covariance matrix Σ. Off-diagonal elements called covariance while the main diagonal elements of the matrix called the variances of the individual variables. There are two useful procedures to solve this problem given by Alt and Smith (1998).

The first procedure is a direct extension of the univariate S^2 control chart. The method is same to repeated test of the importance of the hypothesis that the process covariance matrix is equal to a particular matrix of constant Σ.

The second approach is based on the sample generalized variances, $|S|$. This statistic, which is the determinant of the sample covariance matrix, is a widely used measure of multivariate dispersion. The $|S|$ chart, as presented by Montgomery (2006) have dependent on the mean variance of $|S|$ that is, $E(|S|)$ and $V(|S|)$ and the property that most of the probability distribution of $|S|$ is contained in the interval: $E(|S|) \pm 3\sqrt{V(|S|)}$ as follows as:

$$E(|S|) = b_1 |\Sigma| \tag{2.20}$$

and

$$V(|S|) = b_2 |\Sigma|^2 \tag{2.21}$$

where

$$b_1 = \frac{1}{(n-1)^p} \prod_{i=1}^{p} (n-i) \tag{2.22}$$

and

$$b_2 = \frac{1}{(n-1)^{2p}} \prod_{i=1}^{p}(n-i)\left[\prod_{j=1}^{p}(n-j+2) - \prod_{j=1}^{P}(n-j)\right] \quad (2.23)$$

p = variable,

So, the parameters of the control chart for generalized variance (GV) $|S|$ would be:

$$\left.\begin{aligned} UCL &= |\Sigma|\left(b_1 + 3b_2^{1/2}\right) \\ CL &= b_1|\Sigma| \\ LCL &= |\Sigma|\left(b_1 - 3b_2^{1/2}\right) \end{aligned}\right\} \quad (2.24)$$

The lower control limit in equation (2.24) can replaced with zero if the calculated value is less than zero. Usually in practice, Σ could be estimate by a sample covariance matrix S, based on the analysis of preliminary samples. Accordingly, we should replace $|\Sigma|$ in equation (2.24) by $\frac{|S|}{b_1}$. Since equation (2.20) has shown that $\frac{|S|}{b_1}$ is an unbiased estimator of $|\Sigma|$.

2.6.3 Vector Variance (VV) Chart.

One more step in the multivariate variability measure is to trace of Σ^2. The trace is the sum of all diagonal elements Σ^2 and is called vector variance (VV). The VV is introduced to analyze a set of random vectors (Escoufier, 1973) . The difference GV and VV is GV is the product of all eigenvalues of Σ while VV is the sum of the squares of all elements on the diagonal of the covariance matrix, $Tr(\Sigma^2)$. Both are monotonically increasing function of their eigenvalues. However, their properties are different. The greater the value of VV, the spread of vector min while shrinking the VV, he