# CORE NUCLEUS POLARIZATION IN LAMBDA ($\Lambda$) HYPERNUCLEI



**ZALIMAN BIN SAULI**

**UNIVERSITI MALAYSIA PERLIS**

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# CORE NUCLEUS POLARIZATION IN LAMBDA ($\Lambda$) HYPERNUCLEI



**By**

**ZALIMAN BIN SAULI**
**(0740110208)**

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**UNIVERSITI MALAYSIA PERLIS**

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# TABLE OF CONTENTS

| | | Page |
|---|---|---|
| **DECLARATION OF THESIS** | | ii |
| **ACKNOWLEDGEMENT** | | iii |
| **TABLE OF CONTENTS** | | iv |
| **LIST OF TABLES** | | vi |
| **LIST OF FIGURES** | | vii |
| **LIST OF SYMBOLS** | | ix |
| **LIST OF ABBREVIATIONS** | | xii |
| **ABSTRAK** | | xiii |
| **ABSTRACT** | | xiv |
| **CHAPTER ONE : INTRODUCTION** | | **1 - 7** |
| 1.1 | Overview | 1 |
| 1.2 | Motivation | 3 |
| 1.3 | Objectives and Scope | 5 |
| **CHAPTER TWO : FORMULATION** | | **8 - 45** |
| 2.1 | Thomas-Fermi Model of the Nucleus | 8 |
| 2.2 | The Hamiltonian | 15 |
| | 2.2.1 Nucleon-Nucleon Potential | 16 |
| | 2.2.2 Nucleon-Nucleon-Nucleon Potential | 22 |

2.2.3 Lambda-Nucleon Potential 24

2.2.4 Lambda-Nucleon-Nucleon Potential 27

2.3 Lambda-Binding To Nuclear Matter 29

2.4 *s*-Shell Hypernuclei 39

**CHAPTER THREE : DETERMINATION OF NUCLEAR PARAMETERS AND RESULTS FOR NUCLEI 46 - 62**

**CHAPTER FOUR : HYPERNUCLEI 63 - 72**

4.1 *Λp* Scattering, *s*-Shell Hypernuclei and *Λ*-binding in nuclear matter 64

4.2 Calculation of $E_{\Lambda}$ and Core polarization 66

**CHAPTER FIVE : CORE POLARIZATION RESULTS AND DISCUSSION 73 - 92**

**CHAPTER SIX : FUTURE OUTLOOK AND CONCLUSIONS 93 - 95**

**REFERENCES 96-105**

**LIST OF PUBLICATIONS 106**

# LIST OF TABLES


| Table | | Page |
|---|---|---|
| 2.1 | *NN* Potential Parameters in *MeV* | 45 |
| 3.1 | The parameter values for various fits to binding energies and *rms* radii for type I energy functional. For details, see text. | 53 |
| 3.2 | Binding Energy per nucleon for 32 nuclei for type I energy functional. The experimental values are from reference (Audi, Wapstra & Thibault, 2003). All numbers are in *MeV*. | 54 |
| 3.3 | The experimental *rms* radii are taken from references (Nadjakov, Marinova & Gangrsky, 1994), error bars in square brackets and (H. Vries, Jager & C. Vries, 1986), error bar in parenthesis. The theoretical point *rms* radii are folded with the proton form factor. All *rms* radii are in *fm*. Calculations are for type I energy functional. | 55 |
| 3.4 | The parameter values for various fits to binding energies and *rms* radii for type II energy functional. Note the very large values of $\chi^2$ for B1'. For details, see text. | 56 |
| 3.5 | Same as in Table 3.2. Calculations are for type II energy functional. | 57 |
| 3.6 | Same as in Table 3.3. Calculations are for type II energy functional. | 58 |
| 4.1 | Comparison of calculated and experimental binding energies of *s*-Shell hypernuclei. Rows two and three give the average values of ${}^{4}_{\Lambda}H$ and ${}^{4}_{\Lambda}He$. | 69 |
| 5.1 | Results for a few $\Lambda\Lambda$-hypernuclei demonstrating the core polarization effects. | 81 |

# LIST OF FIGURES


| Figure | | Page |
|---|---|---|
| 3.1 | Energy of Nuclear matter as a function of the density $\rho$ in $fm^{-3}$ for four realistic Hamiltonians. See text for further explanation. The solid curve is the result of our calculations, Equation (2.4) for $M$ = -4.035 *MeV*, $K$ = 250 *MeV* and $\rho_0$ = 0.160 $fm^{-3}$. | 59 |
| 3.2 | The effective compression modulus $K_A$ as a function of $K$ for all the 32 nuclei for $\alpha$ = 1 Figure 3.2a, $\alpha$ = 1/3 Figure 3.2b, and $\alpha$ = 2.0 Figure 3.2c. The filled and open circles are the values of $K_A$ as a function of $K$ for $^{208}Pb$ and $^{122}Sn$ computed by partially mimicking the various effective interactions (see text for details). The lines represent the best fit. | 60 |
| 3.3 | The curves represent the equation of state of symmetric nuclear matter as a function of density $\rho$ for various $K$. | 61 |
| 3.4 | $K_l$ as a function of $\rho$ for various values of $K$. | 62 |
| 4.1 | The solid curve represents the $\Lambda$ binding to nuclear matter $-D_\Lambda$ | 70 |
| 4.2 | The effective mass of $\Lambda$ as a function of density $\rho$. | 71 |
| 4.3 | $\Lambda$ single particle energies ($B_\Lambda$) as a function of mass number for different angular momentum states. The experimental points are from (Hashimoto & Tamura, 2006). The dashed curves represent the results of the calculation as explained in the text. | 72 |
| 5.1 | Polarization Energy in *MeV* as a function of mass number for type I energy functional for various values of $K$. | 82 |
| 5.2 | Change in *rms* radii as a function of mass number for type I energy functional for various values of $K$. | 83 |
| 5.3 | Polarization energy in *MeV* as a function of mass number with $D_\Lambda$= 30 $\rho/\rho_0$ for type I energy functional. | 84 |
| 5.4 | Changes in *rms* radii as a function of mass number with $D_\Lambda$ = 30 $\rho/\rho_0$ for type I functional. | 85 |

5.5 The $\Lambda$ wave function in *s*-state for saturating and non-saturating $D(\rho)$ for type I functional. 86

5.6 The effective compression modulus $K_A$ in *MeV* as a function of the mass number for $K = 250$ *MeV*. 87

5.7 Polarization Energy in *MeV* as a function of mass number for type II energy functional for various values of *K*. 88

5.8 Change in *rms* radii as a function of mass number for type II energy functional for various values of *K*. 89

5.9 Polarization energies as a function of mass number *A* for type II density functional with $D(\rho) = 30\ \rho/\rho_0$ for various values of *K*. 90

5.10 Changes in *rms* radii as a function of mass number *A* for type II density functional with $D(\rho) = 30\ \rho/\rho_0$ for various values of *K*. 91

5.11 The nucleon single particle densities for the nucleus $^{40}Ca$ (solid line) and the hypernucleus $^{42}_{\Lambda\Lambda}Ca$ (dashed line). It may be seen that the presence of $\Lambda$s causes flow of matter from the interior to the exterior region. 92

# LIST OF SYMBOLS

| | |
|---|---|
| $\Lambda$ | Lambda |
| $N$ | Nucleon |
| $\Lambda\,\Lambda$ | Lambda-Lambda |
| $\Lambda N$ | Lambda-Nucleon |
| $\Lambda NN$ | Lambda-Nucleon-Nucleon |
| $\Lambda p$ | Lambda proton |
| $NN$ | Nucleon-Nucleon |
| $NNN$ | Nucleon-Nucleon-Nucleon |
| $\gamma$ | Gamma ray |
| $\pi$ | Pion |
| $\pi^{+}$ | Pion plus |
| $\pi^{\pm}$ | Pion plus minus |
| $\hbar$ | Planck's constant |
| $\mu$ | Reduce mass |
| $\chi^2$ | Chi square |
| $\rho$ | Density of the core nucleus |
| $\rho_m$ | Density at which $\Lambda$-binding to nuclear matter is maximum |
| $\rho_o$ | Nuclear matter equilibrium density |
| $\rho_n(r)$ | Neutron density |
| $\rho_p(r)$ | Proton density |
| $\rho(r)$ | Total nucleon density |

| | |
|---|---|
| $\vec{\sigma}$ | Pauli spin operator |
| $\vec{\tau}$ | Isospin operator |
| $\varepsilon(\rho)$ | Equation of state of symmetric nuclear matter |
| $\sum$ | Sigma hypernuclei |
| $\Xi$ | Xi hypernuclei |
| $\Psi$ | Wave function |
| $a_{\rho}$ | Coefficient of the gradient term |
| $a_{sym}(\rho)$ | Symmetry energy as a function of density |
| $a_{sym}$ | Symmetry energy |
| $a_{pair}$ | Coefficient of the pairing term |
| $A$ | Total mass numbers |
| $c$ | Cut-off parameter |
| $D(\rho)$ | $\Lambda$-binding to nuclear matter |
| $e$ | Electron |
| $E[\rho]$ | Energy density |
| $E_{\Lambda}[\rho]$ | Sum of the kinetic and potential energies of the $\Lambda$ |
| ${}^{A-1}\hat{E}[\rho]$ | Energy of the nucleus in the presence of $\Lambda$ |
| $fm$ | Fermi (Femtometer) |
| $H$ | Hamiltonian |
| $H_{\Lambda}$ | Lambda Hamiltonian |
| $K$ | Compression modulus |
| $K_{A}$ | Effective compression modulus |
| $K^{+}$ | Kaon plus |

| | |
|---|---|
| $K^-$ | Kaon minus |
| $l$ | Angular momentum quantum number |
| $m_i$ | Mass of the nucleon |
| $m_\Lambda$ | Mass of the $\Lambda$ particle |
| $MeV$ | Mega electron volt |
| $n$ | Neutron symbol |
| $N_{n(p)}$ | Normalization constants for the neutrons and protons |
| $p$ | Proton |
| $P_x$ | Majorana space-exchange |
| $R$ | Radius parameter |
| $R_{n(p)}$ | Radius parameters for neutrons and protons |
| $t$ | Surface thickness |
| $t_{n(p)}$ | Surface thickness parameters for neutrons and protons |
| $T$ | Isospin |
| $T_\mu(r)$ | Tensor function |
| $u_v$ | Volume term |
| $V_{ij}$ | Two-nucleon $NN$ potentials |
| $V_{ijk}$ | Three-nucleon $NNN$ potentials |
| $V_{ijk}^{FM}$ | Two-pion exchange part of Fujita and Miyazawa |
| $V_{i\Lambda}$ | Two-body $\Lambda N$ potentials |
| $V_{ij\Lambda}$ | Three-body $\Lambda NN$ potentials |
| $W(r)$ | Woods-Saxon function |
| $Y_\mu(r)$ | Yukawa function |

## LIST OF ABBREVIATIONS

| | |
|---|---|
| *B.E./nucleon* | Binding energies per nucleon |
| BNL | Brookhaven National Laboratory |
| CERN | European Organization for Nuclear Research |
| EM | Electromagnetic |
| FHNC | Fermi-Hypernetted-Chain |
| KEK | High Energy Accelerator Research Organization |
| OPE | One-pion-exchange |
| *rms* | root mean square |
| *s.p.* | single particle |
| TPE | Two-pion exchange |

# POLARISASI TERAS NUKLEUS DALAM

# LAMBDA ($\Lambda$) HYPERNUCLEI

## ABSTRAK

Tindak balas teras nukleus kepada $\Lambda$ dalam hypernucleus dikaji dengan penghampiran kepadatan tempatan. Ini mengeluarkan tenaga dan jejari nukleus teras serta $\Lambda$-tenaga zarah tunggal (*s.p.*) yang baik. Kesan polarisasi $\Lambda$ bergantung kepada tindakbalas teras melalui "kesan" mampatan modulus $K_A$ nukleus. Untuk kelas tertentu tenaga berfungsi, $K_A$ didapati hampir bebas daripada modulus mampatan $K$ nuklear yang tak terhingga. Ini sesungguhnya adalah satu hasil yang mengejutkan, dan bertentangan dengan pengiraan Hartree-Fock dengan interaksi yang berkesan. Sebab-sebab perbezaan ini dikaji dengan teliti. Kami menganggap nilai-nilai $K$ dalam julat ≈ 100-400 *MeV*. Di samping itu, kesan-kesan polarisasi juga bergantung secara kritikal pada $D(\rho)$, mengikat $\Lambda$ dalam hal nuklear pada ketumpatan $\rho$. Untuk hanya satu daya langsung $\Lambda N$: $D \propto \rho$ dan pengecutan nukleus teras membawa kepada polarisasi teras yang secara relatifnya lebih besar. Walau bagaimanapun, untuk "saturating" $D(\rho)$ (dengan maksimum pada $\rho_m < \rho_0$, di mana $\rho_0$ adalah keseimbangan ketumpatan nuklear), yang diperlukan untuk menetapkan *s.p.* ikatan tenaga hypernuclei *s*-Shell dan data berselerak tenaga rendah $\Lambda_p$, dan yang terhasil daripada daya $\Lambda N$ (termasuk pertukaran) dan daya $\Lambda NN$, mungkin terdapat pengembangan nukleus dengan nucleons yang mengalir dari kawasan pedalaman ke permukaan. Ini ditunjukkan untuk mengurangkan kesan-kesan polarisasi teras ketara ($\rho_m$ dalam kejiranan $\rho_0$). Perubahan yang terhasil dalam punca kuasa dua min jejari dan tenaga teras bergantung ke atas $A$, tetapi kebanyakannya sangat kecil, mewajarkannya untuk pengabaian. Kerja-kerja sekarang ini menunjukkan bahawa $\Lambda$ boleh digunakan sebagai alat yang boleh dipercayai untuk menyiasat sifat-sifat nukleus.

# CORE NUCLEUS POLARIZATION IN LAMBDA ($\Lambda$)

# HYPERNUCLEI

## ABSTRACT

The response of the core nucleus to the $\Lambda$ in a hypernucleus is studied with a local density approximation. This reproduces the energies and radii of the core nuclei as well as the $\Lambda$-single particle (*s.p.*) energies quite well. The polarizing effect of the $\Lambda$ depends on the core response through an "effective" compression modulus $K_A$ of the nucleus. For certain class of energy functional, $K_A$ is found to be almost independent of the compression modulus $K$ of the infinite nuclear matter. This indeed is a surprising result, and at variance with the Hartree-Fock calculations with effective interactions. Reasons for this discrepancy were carefully examined, by considering values of $K$ in the range $\approx$ 100-400 *MeV*. Furthermore, the polarizing effects also depend critically on $D(\rho)$, the $\Lambda$ binding in nuclear matter at density $\rho$. For only a direct $\Lambda N$ force: $D \propto \rho$ and the core nucleus contracts giving rise to a relatively larger core polarization. However, for a "saturating" $D(\rho)$ (with a maximum at $\rho_m < \rho_0$, where $\rho_0$ is the nuclear matter equilibrium density), which is required to fit the *s.p.* data, the s-Shell hypernuclei binding energies and the low energy $\Lambda p$ scattering data, and which results from a $\Lambda N$ force (including exchange) and $\Lambda NN$ forces, there may be an expansion of the nucleus with nucleons flowing from the interior to the surface. This is shown to reduce the core polarization effects substantially (for $\rho_m$ in the neighborhood of $\rho_0$). The resulting changes in root mean square radius and core energy depend on $A$, but are mostly very small, justifying their general neglect. The present work thus demonstrates that $\Lambda$ can be used as a reliable tool to probe the properties of nuclei.

# CHAPTER 1

# INTRODUCTION

## 1.1 Overview

The study of hypernuclear is fundamental for improving our knowledge on the strange particle-nucleus interaction that in turn is essential to extend the description of the structure of baryon-baryon interactions and on the properties of the nucleus itself. The first hypernucleus event was reported more than fifty years ago (Danysz & Pniewski, 1953). Since then substantial progress has been made in the experiment as well as in the theoretical areas. In the first two decades, emulsion experiments provided a unique source of information on hypernuclei. This provided valuable information regarding the binding energies of $\Lambda$ hyperon, particularly in the light hypernuclei and their weak decay rates. Substantial progress has been made since then. Combining the data of previous experiments (Davis & Pniewski, 1986) including the Bubble Chamber experiments, the $K^-$ stopped and in-flight $(K^-, \pi^\pm)$ reaction and the associated pair-production $(\pi^+, K^+)$ (Bertini et al. 1980) we have now more than 30 well-established hypernuclei. Experiments carried out at CERN (Brückner, 1975, 1976, 1978; Bertini, 1979, 1981; Povh, 1981; Bedjidian et al. 1980), Brookhaven (BNL) (Chrien et al. 1979; May et al. 1983; May, 1982) and KEK (Yamazaki & Ishikawa, 1982) which gave new data on excited state energies, production intensities and level widths for a number of hypernuclei. These experiments utilized the $(K^-, \pi^\pm)$ reactions. The new data prompted large number of theoretical studies to explain the data and decipher information on baryon-baryon interaction in the strange sector. However, considerable effort is required to make further prediction pertaining to many aspects of hypernuclear physics. The present work is mainly concerned with the use of *Λ* particle to probe the structure of core nuclei. In general, studies of hypernuclei provide important information for understanding the hyperon-nuclear *ΛN* and *ΛNN* interaction.

The lightest hyperon is the neutral *Λ* with strangeness, $S$ = -1 and spin = $\frac{1}{2}$. The *Λ* particle bound to a nucleus has a mass of 1115.6 *MeV* which is about 1.2 times that of a nucleon. In the baryon hypernuclear picture, the *Λ* is distinguishable from the nucleon, so it is allowed to occupy any single particle orbital in the nucleus even taken up by nucleons, without the constrains of the Pauli principle. This single particle structure is observed even in the 1*s* orbit in heavy nuclei. The *Λ* thus provides one of the best examples of single particle shell structure in nuclear physics.

It is now well established that *Λ* forms bound state with all stable nuclei as well as with some nuclei which are normally not particle-stable. These *Λ*-nucleus systems which are referred to as *Λ* hypernuclei are symbolically represented as ${}^{A}_{\Lambda}Z$, where '*Z*' denotes the nuclear charge and '*A*' is the total mass number.

In addition to *Λ* hypernuclei we also have Σ and Ξ hypernuclei but these are not studied in the present work. The additional degree of freedom, namely the strangeness, in all these hypernuclei constitutes a rich and important extension of nuclear physics in the area of hadronic interaction and structure studies.

## 1.2 Motivation

The theoretical study of hypernuclei has focused strongly on learning about the strong and weak hyperon-nuclear interactions (Dalitz, Herndon & Tang, 1972; Gal, 1975; Bodmer & Usmani, 1988). In particular, we have some reasonable knowledge of the strong $\Lambda N$, $\Lambda NN$ and $\Lambda\Lambda$ forces (Bodmer, Usmani & Carlson, 1984; Bodmer & Usmani, 1986; Bodmer & Usmani, 1987; Usmani, Bodmer, & Sharama, 2004; Sinha, Usmani & Taib, 2002; Usmani & Bodmer, 1999; Rijken & Yamamoto, 2006), although there is much more to be learnt. However, there has been the long expressed hope that if the hyperon-nuclear interaction is reasonably well known, one may use the hyperon, in particular the $\Lambda$, to probe the structure of core nuclei. This hope arises from the consideration that the $\Lambda$, being distinguishable, can occupy any state in the nucleus. Also, the lifetime of $\Lambda$-hypernuclei is of the order of $10^{-10}$ seconds, these systems can then be regarded as stable on the strong nuclear time scale. Thus one may address such questions as the effect of $\Lambda$ on the moment of inertia and on rotational bands of the core nucleus, and in general consider the response of nuclei to the presence of a $\Lambda$. It may also be possible in future that $\Lambda$ single particle energies in higher angular momentum states in heavy nuclei may yield some information about nuclear surface properties. In the present work, we study the effect of a $\Lambda$ on spherical core nuclei. In particular, we calculate the changes in binding energy (core polarization energy) and of the root mean square (*rms*) radii of the core nuclei due to presence of the $\Lambda$.

The presence of a $\Lambda$ in hypernucleus causes a compression or dilation of the core nucleus depending upon the nature of the $\Lambda$-nuclear interactions (Lanskoĭ &

Tret'yakova, 1989; Ho & Volkov, 1969; Ho & Volkov, 1970; Žofka & Sotona, 1978; Rayet, 1981). The relationship between the incompressibility and the change in the core size as well as the polarization energy due to presence of *Λ* has been extensively studied mainly within the Hartree-Fock approximation using Skyrme and finite range effective interactions (Lanskoǐ & Tret'yakova, 1989; Ho & Volkov, 1969; Ho & Volkov, 1970; Žofka & Sotona, 1978; Rayet, 1981; Žofka, 1980; Žofka, 1982; Bassichis & Gal, 1970; Rayet, 1976; Lanskoy & Yamamoto, 1997; Lanskoy, 1998). A few studies pertain to model calculations (Lanskoǐ & Tret'yakova, 1989; Feshbach, 1976) based upon qualitative considerations. The polarization energies are found to be 0.1 to 1.2 $MeV$ in the range $16 \leq A \leq 40$ and increases or stays constant with respect to *A* at least within this range (Lanskoǐ & Tret'yakova, 1989; Žofka, 1980). The core polarization energies have also been found to decrease as *K* increases. In one case of ${}^{16}_{\Lambda}O$, it was found to increase with $K$ (Lanskoy, 1998). However, these values of core polarization energies are not small and secondly to our knowledge they are mostly confined to light and medium *A* nuclei. It was demonstrated in (Usmani, Bodmer & Sharama, 2004) that for ${}^{6}_{\Lambda\Lambda}He$, the core polarization effect magnifies in double hypernuclei where the polarization energies increase roughly by a factor of 3 or more (Lanskoy, 1998; Hiyama, Kamimura, Motoba, Yamada & Yamamoto, 2002). Their inclusion assumes importance in cluster model calculations where in a few cases a rigid core approximation has been used. For a consistent treatment of hypernuclei it is thus desirable that we study core polarization in greater detail. The only empirical knowledge about core-polarization comes from a $\gamma$ transition and is limited to contraction of the ${}^{6}Li$ core nucleus in ${}^{7}_{\Lambda}Li$ (Tanida et al., 2001). But this represents a very special situation where the *Λ* probably shrinks the *rms* radius of the loosely bound *p*-shell nucleons by a large amount. We shall very briefly comment on this at a later stage.

## 1.3 Objectives and Scope

In the present study, the main objective is develop an extended Thomas-Fermi theory using a local density approach ("Thomas Fermi" model for short) to calculate binding energies, root mean square radii, density distributions for nuclei and hypernuclei and demonstrate that for "realistic" $\Lambda$-nuclear interactions, the core polarization effects are in general very small, both the polarizing energies as well as the change in *rms* radii. The smallness of the core polarization effects on radii was also pointed out by Rayet (Rayet, 1981) who inferred that presence of a repulsive three-body $\Lambda NN$ force may turn a contraction into dilation for ${}^{16}_{\Lambda}O$ . A dilation of the core nucleus in presence of $\Lambda$ has also been found (Lanskoĭ & Tret'yakova, 1989), but in most of the studies contraction is a preferred conclusion (Lanskoĭ & Tret'yakova, 1989; Ho & Volkov, 1969; Ho & Volkov, 1970; Žofka & Sotona, 1978; Rayet, 1981; Žofka, 1980; Žofka, 1982; Bassichis & Gal, 1970; Rayet, 1976; Lanskoy & Yamamoto, 1997; Lanskoy, 1998; Feshbach, 1976; Hiyama, Kamimura, Motoba, Yamada, & Yamamoto, 2002; Tanida et al., 2001).



For a direct $\Lambda N$ interaction, our results are in line with earlier studies. However, there are important major differences. We find that the behavior of core polarization depends upon an "effective compression modulus", $K_A$ (to be defined later) of a particular nucleus and on the structure of nuclear surface. It does not depend directly on the compression modulus $K$ of infinite nuclear matter. $K_A$ is commonly used in the calculation of the energies of the giant monopole resonances which correspond to the “breathing mode” of the nucleus (Blaizot, 1980). Our nuclear Hamiltonian, or the energy density functional does not depend on Skyrme or any other *NN* interaction, but is rather determined by expanding the energy per nucleon of nuclear matter around the equilibrium density by means of Taylor series and then adopting a purely phenomenological approach. In particular, we find that for certain class of energy functional, $K_A$ depends on the compression modulus $K$ of the infinite nuclear matter rather weakly and decreases gradually with increasing $K$ in the range 100 to 300 *MeV* and then starts rising slowly. This indeed is a surprising result. Thus the relationship between the compressibility and the polarization of the core nucleus is not as simple as one might have expected it form the earlier Hartree-Fock studies which employ either zero-range Skyrme or finite range effective interactions. In the Hartree-Fock scheme $K_A$ is found to be proportional to $K$ (Blaizot, 1980; Blaizot, Berger, Dechargé & Girod, 1995) and has a strong dependence on it. We discuss in detail the reasons for this paradox by partially emulating finite range as well as the zero range Skyrme interactions with in our formalism.



We take into account the differences between the neutron and proton densities explicitly arising out of the neutron-proton imbalance and the presence of Coulomb forces in nuclei. The nuclei (hypernuclei) considered range from $^{10}B$ ($^{11}_{\Lambda}B$) to $^{243}Am$ ($^{244}_{\Lambda}Am$), a total of 32 nuclei. Our local density approach gives a good description of the static properties of nuclei and hypernuclei, such as the binding energies and *rms* radii. The approach uses the variational principle which minimizes the energy of the nucleus (hypernucleus) with respect to changes in neutron and proton densities. Thus *rms* charge radii, nuclear surface diffuseness, total binding energies and other nuclear properties are an outcome of the theory.

The energy of the hypernucleus is the sum of the energy of the core nucleus and that of the *Λ*:

$$ {}_{\Lambda}^{A}E[\rho] = {}^{A-1}\hat{E}[\rho] + E_{\Lambda}[\rho], \tag{1.1} $$

where $\rho$ is the density of the core nucleus; $E_{\Lambda}[\rho]$ is the sum of the kinetic and potential energies of the *Λ* moving in the potential generated by the *Λ*-nuclear interactions. ${}^{A-1}\hat{E}[\rho]({}^{A-1}E[\rho])$ is the energy of the nucleus in the presence (absence) of the *Λ*. The square brackets indicate that the energies are functional of *ρ*. The calculation of ${}^{A-1}E[\rho]$ is described in Chapter 2, and that of $E_{\Lambda}[\rho]$ (with the *Λ* in $\ell \le 4$ states) is described in reference (Usmani & Bodmer, 1999; Usmani, Sami & Bodmer, 1992; Usmani, Sami & Bodmer, 1994) and briefly in Chapter 4. This is followed in Chapter 3 by the determination of nuclear parameters. In Chapter 4 we also describe the calculation of ${}_{\Lambda}^{A}E[\rho]$ including the polarizing effect of the *Λ*. Chapter 5 presents and discusses our results for the polarizing energy and related questions. Chapter 6 is the conclusions of the finding in this research.

# CHAPTER 2

# FORMULATION

## 2.1 Thomas Fermi Model of the Nucleus

Our model is phenomenological and generally has been in use for a long time (Myers & Swiatecki, 1969; Myers & Swiatecki, 1974; Treiner & Krivine, 1986; Dabrowski & Kohler, 1989) for early version and reference (Centelles, Leboeuf, Monastra, Roccia, Schuck & Viñas, 2006; Centelles, Schuck & Viñas, 2007) for more recent elaborate versions. For our purposes this model gives a good description of nuclei and adequately describes the nuclear response of the $\Lambda$.

The energy ${}^{A-1}\hat{E}[\rho]$ or ${}^{A-1}E[\rho]$ is an integral of an energy density which accounts for the volume, surface, asymmetry energies plus coulomb and pairing terms:

$$ {}^{A-1}E[\rho] = \int \left[ \varepsilon(\rho) + \frac{\hbar^2}{72m}\left(\frac{\nabla\rho}{\rho}\right)^2 + \frac{\hbar^2}{6m}\nabla^2\rho + a_\rho \frac{(\nabla\rho)^2}{\rho} + a_{sym}(\rho)\left(\frac{\rho_n - \rho_p}{\rho}\right)^2 \right] \rho \, d\vec{r} $$

$$ + \, Coulomb \; + Pairing \;\; , \tag{2.1} $$

where $\rho_n(r)$ and $\rho_p(r)$ are respectively the neutron and proton densities, and $\rho(r)$ is the total nucleon density; $\rho(r) = \rho_n(r) + \rho_p(r)$. We ignore shell and deformation effects which have little relevance for the present investigation. In expression (2.1), the term $\varepsilon(\rho)$ represents the equation of state of symmetric nuclear matter, i.e., the binding energy per nucleon as a function of nuclear matter density. The terms $(\nabla\rho)^2/\rho$ and $(\nabla\rho)^2/\rho^2$ are essential for the surface properties. To a very good approximation $a_{sym}(\rho)$ can be considered as independent of $\rho$ (Bombaci & Lombardo, 1991). The parameters $a_{sym}$ and $a_\rho$ are determined by fitting ${}^{A-1}E$ to the experimental binding energies and *rms* radii of nuclei as described later. For the coulomb energy we use

$$Coulomb \;=\; \frac{1}{2}e^2\int\frac{\rho_p(\vec{r}_1)\rho_p(\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|}d\vec{r}_1 d\vec{r}_2 - \frac{3}{4}\left(\frac{3}{\pi}\right)^{1/3} e^2\int\rho_p^{4/3}(r)d\vec{r}, \tag{2.2}$$

where the second term on the right hand side is an approximation to the exchange part of the coulomb energy. For the pairing term we employ,

$$Pairing \;=\; -a_{pair}\frac{(-1)^Z+(-1)^N}{(N+Z)^{3/4}} \tag{2.3}$$

Though, we have included the small pairing energy term, but it is not expected to play significant role in the present study.

For $\varepsilon(\rho)$ in Equation (2.1) one may utilize a functional from the results of nuclear matter calculations using some effective interaction like Skyrme (Beiner, Flocard, Giai & Quentin, 1975) or Gogny (Berger, Girod & Gogny, 1991; 1989) types, or the one which imitates a realistic Hamiltonian (Wiringa, Stoks & Schiavilla, 1995; Pudliner, Pandharipande, Carlson & Wiringa, 1995; Wiringa, Fiks & Fabrochini, 1988; Akmal, Pandharipande & Ravenhall, 1998). However, this ties us to a specific form of the interaction. Therefore, in the present study, we adopt a more general approach in which the different values of the parameters of $\varepsilon(\rho)$ could possibly emulate, at least partly, the various diverse interactions. Making a Taylor series expansion of $\varepsilon(\rho)$ around $\rho = \rho_0$, and since at saturation $d\varepsilon / d\rho |_{\rho_0} = 0$, leads to the following:

$$\varepsilon(\rho_{\geq}) = -u_v + \frac{K}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + M\left(\frac{\rho-\rho_0}{\rho_0}\right)^3, \qquad \text{for } \rho \geq \rho_x \qquad (2.4a)$$

$$\varepsilon(\rho_{\leq}) = A\rho + B\rho^2 + C\rho^3 + D\rho^4 + \frac{3\hbar^2(3\pi^2)^{2/3}}{10 m_N \rho}\left(\rho_n^{5/3} + \rho_p^{5/3}\right). \quad \text{for } \rho \leq \rho_x \qquad (2.4b)$$

In Equation (2.4a), the parameter $M$ is a measure of deviation from parabola in the vicinity of the saturation density $\rho_0$, which is related to an asymmetry in the saturation curve. We may have an idea of the values of $M$ from the calculation of nuclear matter equation of state using realistic Hamiltonians (Wiringa, Stoks & Schiavilla, 1995; Pudliner, Pandharipande, Carlson & Wiringa, 1995; Wiringa, Fiks & Fabrochini, 1988; Akmal, Pandharipande & Ravenhall, 1998). Higher densities $\rho > 0.20\ fm^{-3}$ are irrelevant in the present study as they are not accessible by normal nuclei or hypernuclei. Thus the values of $M$ generally refer to low values of $\rho$. The density $\rho_x$ is a parameter between 0 and $\rho_0$ to be determined as described later. The terms containing $\rho^{5/3}$ are the neutron and proton single particle kinetic energies.