International Journal of Nanoelectronics and Materials
Volume 12, No. 1, Jan 2019 [67-74]

Implementation of Sawtooth Wavelet Thresholding for Noise Cancellation in One Dimensional Signal

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Received 28 June 2018; Revised 15 August 2018; Accepted 27 September 2018

ABSTRACT

Wavelet families have different statistical characteristics and specifications which give them a different response against the same signal or image when they are used for a certain task such as signal denoising. Therefore, a comparison evaluation study using new proposed procedure is required to obtain the optimal results when wavelet analysis tool is used to remove the noise from a synthetic signal. In this work, a sawtooth wavelet thresholding method is proposed and evaluated as compared to the other wavelet thresholding methods such as (soft and hard). The main goal of this work is to design and implement a new wavelet thresholding method and evaluate it against other classical wavelet thresholding methods and hence search for the optimal wavelet mother function among the above mentioned families with a suitable level of decomposition followed by a novel thresholding method among the existing methods. This optimal method will be used to shrink the wavelet coefficients and yield an adequate denoised pressure signal prior the transmission. There are different performance indices to establish the comparison and evaluation process for signal denoising; but the most well-known measuring scores are: NMSE (normalized mean square error), ESNR (enhancement of signal to noise ratio), and PDR (percentage root mean squared difference). The obtained results shown the out-performance of the sawtooth wavelet thresholding method against other methods using different measuring scores and hence the conclusion is to suggest the adopting of this proposed wavelet thresholding for 1D signal denoising in future researches.

Keywords: SNR (Signal to Noise Ratio), Cross Correlation, Signal Denoising, Sawtooth Wavelet Thresholding.

1. INTRODUCTION

Wavelets theory represents another way to construct the signal model based on some special signals known as wavelets. Wavelets are short in time extent and alternator oscillating with certain amplitude. Unlike Fourier series that has smooth sinusoid bases, wavelets have asymmetric and irregular bases [1].

Wavelet transform is a good tool for signal analysis and one of its applications is the denoising and compression of signals and images [2]. In this work, a special type of signal is adopted which is synthesis signal for measuring an internal pressure for a certain fluid inside a pipeline [3].

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Measurement for any physical parameter requires a sensing element which is used to convert the required signal to a suitable readable form for the next stage, and this process unfortunately cannot be safely done without an existence of an inherent noise [4]. So, signal denoising is the appropriate suggestion for any measuring process in order to get a precise measurement at the final process, and signal denoising can be achieved via different algorithms such as wiener filtering, spectral subtraction, and wavelets transform [5]. Wavelet transform based signal denoising or compression can be summarized by three steps [6]:

i- Transform the signal from time domain to the wavelet domain such that most of the signal energy concentrated in few wavelet coefficients know as approximation,
ii- Thresholding the detail wavelet coefficients which are considered as a non-required part of the signal (i.e. inherent noise),
iii- Inverse transformed backs the wavelet coefficients (approximations and details) from wavelet domain to the time domain and hence gets the denoised signal.

In this work, a sawtooth wavelet thresholding method is proposed which can be considered as a modification for the well-known thresholding methods (soft and hard). In essence, this modification is based on filter characteristics theory, which adopts the differences between Butterworth and Chebyshiv filters.

In filter theory, Butterworth filter has flat pass band, with ripple free characteristics. In contrast to the Chebyshiv filter which has ripple in the pass band. This idea is developed for the wavelet thresholding function. Since soft and hard thresholding are ripple free in their characteristics, while the proposed new thresholding method has ripple in the pass band like Chebyshiv filter characteristics [7]. But here the proposed ripples take a form of sawtooth signal rather than sinusoids.

2. WAVELET THRESHOLDING RELATED WORK

One of the most interesting points for the researcher in the wavelet thresholding domain is how to improve the thresholding method or the threshold selection rule in order to get an optimal results that minimize certain criteria such as MSE (mean square error), RMSE (root mean square error), NMSE (normalized mean square error), PDR (percentage root mean square difference), or maximizing other performance indices such as PSNR (peak signal to noise ration), and ESNR (enhancement signal to noise ratio) [8].

Some of the past years related work in the development of the thresholding methods can be summarized as follows:

i. Some researchers in 2017 suggested a new hierarchy for wavelet thresholding method that overcome the problem of global threshold by choosing self-adaptive thresholds based on the fact of noise decay rate in the wavelet detail coefficient [9].
ii. Another wavelet thresholding function was proposed in 2018 as a modification for the soft and hard thresholding by augmenting two factors [shaping and scaling] to make it continuously differentiable at all points, and give it maximum adaptability with the input signal under denoising process [10].
iii. Another researchers in 2018 suggested wavelet transform based logarithmic thresholding for de-noising of images, corrupted by noise (during under-sampling in the frequency domain) [11]. The logarithmic shrinkage technique is applied to under-sampled Shepp–Logan Phantom image. The experimental results show 10% enhancement over the traditional thresholding methods in removing different type of noise such as salt and pepper, Gaussian, speckle, and Poisson noises. In addition to that, the experiments show 35% enhancement over the classical methods if wiener filtering

with median threshold combined with the logarithmic wavelet thresholding method [12].

iv. In 2018, researchers proposed a wavelet based estimation to de-noise 1D or 2D signal by evaluating threshold value using FDR, Visu and Top rules. The proposed algorithm was simulated using MATLAB with the performance indices such as Signal to Noise Ratio (SNR) and Mean Square Error (MSE) [13]. From the work analysis, it was observed that the mixed function performs better than all existing functions for 1D and 2D signals in FDR and Visu rules while in top rule, the mixed function performs better only in hard thresholding for both 1D and 2D signals [14].

v. An additional wavelet thresholding method was proposed at 2018, which adopt wavelet thresholding based on noise variance estimation. This method is known as improved thresholding function. The study put forward the strategy of using two-state Gaussian mixture model in order to classify the high-frequency wavelet coefficients in the minimum scale accordingly. The experiment with different test signals shows the suitability of the improved thresholding for electro-mechanical transmission system because it combines the advantages of the soft and hard thresholding methods [15].

Our proposal of wavelet thresholding method is to augment the soft and hard thresholding by sawtooth signal as a ripple in their pass-band region such that new softy-sawtooth and hardy-sawtooth thresholding methods are constructed as shown in Figure 1.

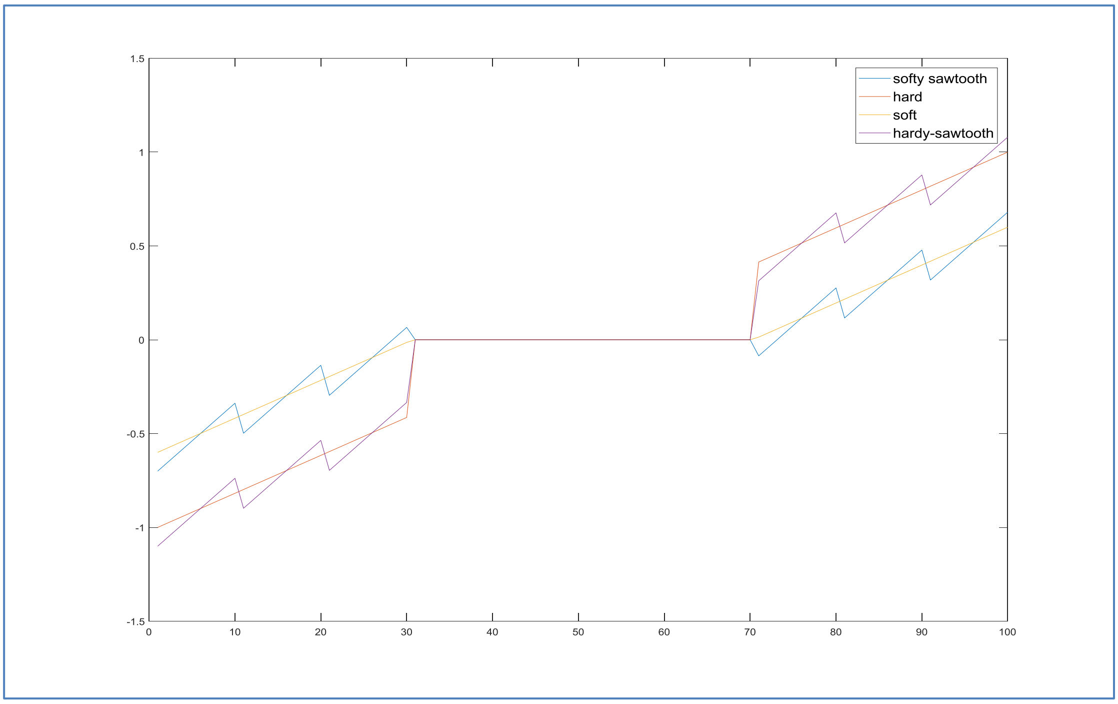

Figure 1. Softy and hardy sawtooth wavelet thresholding methods.

3. EVALUATION PERFORMANCE CRITERIA

There are different performance measures to evaluate the denoising process, and these parameter depend in their evaluation for denoising calculation on the original clean signal and the denoised signal characteristics. Table 1 shows some of these performance measure which is used in the work to evaluate the proposed thresholding method and compare it with the classical methods. In addition, the correlation between the basis and the analyzed signal is an important measure for the success of the analysis process since as high as the cross correlation between the scaling wavelet function with the signal to be analyzed yields the most cumulative energy concentrated in few number of wavelet approximation coefficients, leaving the details wavelet coefficients represent the unwanted noise, which will be shrinkage in the further denoising steps after signal decomposition [16]. The results of correlation between the noisy synthesis pressure signals with different family of wavelet scaling function are shown in Table 2.

Table 1 Different performance indices for evaluation of signal denoising algorithm

SNR	
NMSE	
RMSE	
PDR	

Table 2 Cross correlation between the noisy pressure signal and wavelet scaling functions

type of wavelet scaling function	% cross correlation between wavelet scaling function and pressure signal
biorthogonal 6.8	5.5091
symlet 8	83.5997
coiflet 2	22.4386
discrete meyer	21.8229
reverse biorthogonal 4.4	28.9379

4. PROPOSED MATHEMATICAL MODEL

Wavelet thresholding represent the backbone for the wavelet denoising algorithm. The research area for the development of the thresholding techniques still has a wide area that are worth explored since the first thresholding method proposed in 1995 by Donoho [17]. Researchers have been developing the thresholding method, for example in 2018, augmented, logarithmic, mixed, and improved thresholding methods are suggested for both signal and image denoising. The mathematical equation for the thresholding function which represent the input-output characteristics for this process is described by:

i. Hard thresholding

$$Q_j = \begin{cases} W_j & |W_j| > \lambda \\ 0 & |W_j| \leq \lambda \end{cases}$$

ii. Soft thresholding

$$Q_j = \begin{cases} [sign\ (W_j)(|W_j| - \lambda)] & |W_j| > \lambda \\ 0 & |W_j| \leq \lambda \end{cases}$$

iii Softy-sawtooth thresholding

$$Q_j = \begin{cases} [sign\ (W_j)(|W_j| - \lambda)] \\ +a\ sawtooth(b\pi W_j) & |W_j| > \lambda \\ 0 & |W_j| \leq \lambda \end{cases}$$

iv- Hardy-sawtooth thresholding

$$Q_j = \begin{cases} [W_j]\ + \\ a\ sawtooth(b\pi W_j) & |W_j| > \lambda \\ 0 & |W_j| \leq \lambda \end{cases}$$

5. SIMULATION RESULTS

A synthesis pressure signal with 100 sample is constructed to simulate a pressure inside a pipeline with an estimated leak size of 0.25''. In addition to the simulated pressure, random noise signal is added to the clean pressure signal such that a noisy pressure signal is constructed and become the input to the denoising algorithm. The proposed wavelet thresholding method was simulated and tested using matlab 2017.

Among different wavelet families, and based on the correlation between wavelet mother function and the signal under test, symlet family was selected for the evaluation of the proposed thresholding method and the results was compared with the classical soft and hard thresholding methods as explained in details in Table 3, Table 4, and Table 5.

In Table 3, different symlet wavelets are used to decompose the noisy synthesis pressure signal. The comparison between hard and hardy-sawtooth shows the outperformance of the later upon the former if symelet8 is used for the denoising process when all other parameter are fixed at certain values, such as level 5 and universal threshold selection rule and level dependent estimates of the noise for threshold rescaling principle.

Table 3 Denoising results using different symlet family wavelet mother functions

thresholding method	signal type	level	thrshold selection rule	threshold rescaling
Hard	noisy synthesis pressure signal yn with 0.25 leak size	5	sqtwolog	level-dependent estimates of the noise

wavelet name		sym 10	sym 8	sym 6	sym 4	sym 2
performance	NMSE	0.0256	0.0207	0.0237	0.0263	0.0323
	RMSE	0.1131	0.01058	0.109	0.1146	0.1271
	ESNR	9.8932	**10.2265**	9.8154	9.7792	8.8754
	PDR	0.0518	0.0469	0.0499	0.0525	0.0583

thresholding method	signal type	level	thrshold selection rule	threshold rescaling
Hardy-sawtooth	noisy synthesis pressure signal yn with 0.25 leak size	5	sqtwolog	level-dependent estimates of the noise

wavelet name		sym 10	sym 8	sym 6	sym 4	sym 2
performance	NMSE	0.0186	0.0137	0.0167	0.0193	0.0253
	RMSE	0.0631	0.099	0.059	0.0646	0.0771
	ESNR	10.2932	10.4154	10.2154	10.1792	9.2754
	PDR	0.0218	0.0399	0.0199	0.0225	0.0283

Table 4 summarized the results for the level decomposing selection process, where the levels are varied and the performance was examined using the proposed indices and keeping the other parameters that involved in the denoising algorithm fixed (such as wavelet type, thresholding method, threshold selection rule and threshold rescaling method)**.**

After the selection of symlet8 as wavelet mother function with level 5 as the best decomposition level for denoising under test pressure signal; Table 4 examined different threshold selection rule and compare between them in order to identify the best rule among them.

Table 4 Denoising results using symlet 8 wavelet mother function with different decomposition levels

thresholding method	signal type	wavelet type	shold selection ru	threshold rescaling
Hard	noisy synthesis pressure signal yn with 0.25 leak size	sym8	sqtwolog	level-dependent estimates of the noise

decomposition level		1	2	3	4	5	6	7	8
performance	NMSE	0.1346	0.0729	0.0384	0.0274	0.0207	0.0308	0.1107	0.1339
	RMSE	0.2594	0.1909	0.1386	0.117	0.01058	0.124	0.2352	0.2588
	ESNR	2.6794	5.3431	8.1266	9.5953	**10.2265**	9.0908	3.5298	2.7027
	PDR	0.1189	0.0875	0.0635	0.0536	0.0469	0.0568	0.1078	0.1186

thresholding method	signal type	wavelet type	shold selection ru	threshold rescaling
Hardy-sawtooth	noisy synthesis pressure signal yn with 0.25 leak size	sym8	sqtwolog	level-dependent estimates of the noise

decomposition level		1	2	3	4	5	6	7	8
performance	NMSE	0.1346	0.0729	0.0384	0.0274	0.0137	0.0309	0.1108	0.1338
	RMSE	0.2594	0.1909	0.1386	0.117	0.099	0.1243	0.2354	0.2586
	ESNR	2.6794	5.3431	8.1266	9.5953	10.4154	9.0717	3.5245	2.7063
	PDR	0.1189	0.0875	0.0635	0.0536	0.0399	0.057	0.1079	0.1186

From the results in Table 5, it is clear that the proposed hardy-sawtooth thresholding has larger ESNR (or lower NMSE, RMSE, and threshold (labeled by sqtwolog). In addition, the results also shows the comparison between softy-sawtooth and classical soft thresholding for different threshold selection rule.

Table 5 Denoising results using symlet 8 for wavelet different threshold selection rules

signal type					wavelet type	level	threshold rescaling		
noisy synthesis pressure signal yn with 0.25 leak size					sym8	lev=5	level-dependent estimates of the noise		
thrshold selection rule		huresure		sqtwolog		rigrsure		minimaxi	
thresholding method		Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft
performance	**NMSE**	0.0264	0.0259	0.0207	0.0298	0.0954	0.0286	0.0688	0.0244
	RMSE	0.1181	0.1172	0.1058	0.1252	0.2188	0.1227	0.1865	0.114
	ESNR	9.2934	9.3588	**10.2265**	8.8049	4.0395	8.9716	5.4091	9.5969
	PDR	0.0525	0.0521	0.0469	0.0557	0.0987	0.0546	0.0838	0.0506
thrshold selection rule		huresure		sqtwolog		rigrsure		minimaxi	
thresholding method		Hardy-sawtooth	Softy-sawtooth	Hardy-sawtooth	Softy-sawtooth	Hardy-sawtooth	Softy-sawtooth	Hardy-sawtooth	Softy-sawtooth
performance	**NMSE**	0.0197	0.019	0.0137	0.0231	0.0899	0.0222	0.0624	0.0175
	RMSE	0.1119	0.1104	0.099	0.1186	0.2135	0.1169	0.1803	0.1072
	ESNR	9.4411	9.5468	**10.4154**	8.976	4.1743	9.0895	5.5716	9.7845
	PDR	0.0459	0.0452	0.0399	0.0489	0.0925	0.0482	0.0772	0.0437

6. CONCLUSIONS

The work in this paper shows a systematic procedure in order to obtain the best denoising results using matlab program. Programs for new proposed algorithm had been written to simulate the results for denoising the proposed synthesis pressure signal of a pipeline leaked by 0.25''. The denoising process involved a comparison among different wavelets from symlet family, and choosing the best mother function among them. Then, the procedure continues to select the best decomposing level for the chosen wavelet. The comparison between different threshold selection rule and the results of comparison procedure emphasis on the outperformance of the new proposed thresholding method of hardy-sawtooth upon the classical hard rather than softy-sawtooth and soft thresholding methods using four different performance indices NMSE, RMSE, ESNR and PDR.

REFERENCES

[1] M. Sifuzzaman, M. R. Islam, M. Z. Ali, Journal of Physical Sciences **13** (2009) 121.
[2] S. Mallat, IEEE Trans. Pattern Anal. Machine Intell. **11** July (1989) 674.
[3] S. G. Chang, B. Yu, M. Vetterli, IEEE Trans. Image Processing **9** Sept. (2000) 1532.
[4] M. Lang, H. Guo, J. Odegard, C. Burrus, R. Wells, IEEE Signal Proc. Letters **3** (1996) 10.
[5] Q. Pan, L. Zhang, G. Z. Dai, H. C. Zhang, IEEE Trans. Signal Proc. **47** (1999) 3401.
[6] J. Gao, H. Sultan, J. Hu, W. W. Tung, IEEE Signal Processing Letters **17** (2010) 237.
[7] Y. Xu, J. B. Weaver, D. M. Healy, IEEE Transactions on Image Processing **3** (1994) 747.
[8] B. Bhonsle, N. Dewangan, International journal of emerging trends and technology in computer science (IJETTCS) **2** (2013) 57.
[9] J. Zhang, Q. Zhu, L. Song, J. Wang, Journal of Petroleum Science and Engineering **160** (2018) 433.
[10] X. Xiaobin, Infrared Physics & Technology **88** (2018) 174.
[11] A. Hayat Ullah, U. Ihsan, K. Shafqat Ullah, M. Khan, Wireless Press Communications **98** (2018) 1473.
[12] S. U. Khan, M. K. A. Rahim, I. M. Qureshi, N. A. Murad, International Journal of Electrical and Computer Engineering (IJECE) **7** (2017) 961.
[13] M. Koteswararao, IEEE Sensors Letters **2** (2018) 104.
[14] K. Manjit, International Journal Of Engineering And Computer Science **2** (2013) 2932.
[15] L. Hui, W. Weida, X. Changle, H. Lijin, N. Haizhao, Mechanical Systems and Signal Processing **99** (2018) 30.
[16] X. Y. Wang, X. X. Ou, B. W. Chen, M. Kim, IJDSN **2** (2015) 23.
[17] D. L. Donoho, IEEE Transactions on Information Theory **41** (1995) 613.